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Remarks on Interiors and Closures of Weak Open Sets in Bigeneralized Topological Spaces

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Abstract

We establish the relationships between the interior and closure operators among the μ_{ij}-semiopen, μ_{ij}-preopen, $\alpha\mu_{ij}$-open, $\beta\mu_{ij}$-open sets in bigeneralized topological spaces.
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1 Introduction

The study of generalized topological spaces (briefly GTS) was first initiated by A.Csaszar on 2002 (Császár, 2002). In that, he established the interior and closure operators in GTS. Later in 2010 (Boonpok, 2010)this study is extended to bigeneralized topological spaces (briefly BGTS) by C. Boonpak. In his paper, he found the conception of (m, n)-closed and (m, n)-open sets in BGTS. Also, he introduced $(m, n)g$-regular open, $(m, n)g$-semi open, $(m, n)g$-pre open, $(m, n)g - \alpha$-open . With these definitions, the properties of the weak open sets were studied by A. Jamuna Rani and M. Anees Fathima in (Fathima and Rani, 2019; Rani and Fathima, 2020) and some of its characterizations are also analysed.

The purpose of this paper is to prove the relationships between the interior and closure operators among μ_{ij}-semi open, μ_{ij}-preopen, $\alpha\mu_{ij}$-open and $\beta\mu_{ij}$- open sets in BGTS. Also we establish some of its Characterizations.

2 Preliminares

In this section, We provide some basic definitions and notations which are most essential to understand the subsequent section.

For any non empty set X and $\xi \in p(X)$, ξ is said to be GT (Császár, 2002) if $\phi \in \xi$ and ξ is closed under arbitrary union. Also, the function $\gamma \in \Gamma$ (where Γ denotes the collection of all mappings $\gamma : p(X) \to p(X)$ possessing the property of monotony. ie.,if $A \subset B \Rightarrow \gamma(A) \subset \gamma(B)$) is said to be μ-friendly (Császár, 2006)if $\gamma(A) \cap L \subset \gamma(A \cap L)$ for every $A \subset X$ and $L \in \mu$. If $\gamma \in \Gamma$ and $\mu = \{A \subset X / A \subset \gamma(A)\}$is the family of all γ- open sets, then μ is a GT (Császár, 2002) The pair (X, μ) is a called GTS. In (Sivagami, 2008) the family of all μ-friendly functions is denoted by Γ_4 and (X, γ) is called the γ-space. It is further proved that every γ-space is a quasi-topological space in (Császár, 2008) and all the results established in for γ-spaces are valid for quasi-topological spaces. Also for $\gamma \in \Gamma$, define $\gamma^* : p(X) \to p(X)$ by $\gamma^*(A) = X - \gamma(X - A)$ (Császár, 1997) for every subset A of X.

Let X be a non empty set and μ_1, μ_2 be generalized topologies on X. A triple (X, μ_1, μ_2) is said to be a bigeneralized topological space. Let A be subset of a bigeneralized topological space X. Then the closure of A and the interior of A with respect to μ_m are denoted by $c_{\mu_m}(A)$ and $i_{\mu_m}(A)$ respectively, for $m = 1, 2$. (Boonpok, 2010) A subset A of a bigeneralized topological space(X, μ_1, μ_2) is said to be μ_{ij}-semiopen (Fathima and Rani, 2019)(resp.μ_{ij}-preopen (Rani and Fathima, 2020) $\alpha\mu_{ij}$-open$\beta\mu_{ij}$-open (Jamuna Rani and Anees Fathima, 2021)) if $A \subset c_{\mu_i} i_{\mu_j}(A)$ where $i, j = 1, 2$ and $i \neq j$ (resp. $A \subset i_{\mu_i} c_{\mu_j}(A)$, $A \subset$

$i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$,$A \subset c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$).

Proposition 1.1. (Min, 2009)Let (X, μ) be a generalized topological space. For subsets A and B of X, the following properties holds.

(a) $c_\mu(X - A) = X - i_\mu(A)$ and $i_\mu(X - A) = X - c_\mu(A)$.

(b) If $(X - A) \in \mu$, then $c_\mu(A) = A$ and if $A \in \mu$, then $i_\mu(A) = A$.

(c) If $A \subseteq B$, then $c_\mu(A) \subseteq c_\mu(B)$ and $i_\mu(A) \subseteq i_\mu(B)$.

(d) If $A \subseteq c_\mu(A)$ and $i_\mu(A) \subseteq A$.

(e) $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(i_\mu(A)) = i_\mu(A)$.

Proposition 1.2. (Jamunarani et al., 2010) Let (X, γ) be a γ- space . Then $G \cap c_\gamma(A) \subset c_\gamma(G \cap A)$, for every $A \subset X$ and γ-open set G of X.

Proposition 1.3. Let (X, μ) be a quasi topological space and $A, B \subseteq X$, the following holds.

(a) If A and B are μ-open sets, then $A \cap B$ is μ-open (Sivagami, 2008)

(b) $i_\mu(A \cap B) = i_\mu(A) \cap i_\mu(B)$, for every subsets A and B of X (Császár, 2008)

(c) $c_\mu(A \cup B) = c_\mu(A) \cup c_\mu(B)$, for every subsets A and B of X(Sivagami, 2008)

Proposition 1.4. (Fathima and Rani, 2019) Let (X, μ) be a generalized topological space. Let A be a subset of X. Then the following hold.

(a) $c_{\sigma_{ij}}(A)$ is the smallest μ_{ij}-semi closed set containing A.

(b) A is μ_{ij}-semi closed if and only if $A = c_{\sigma_{ij}}(A)$.

(c) $x \in c_{\sigma_{ij}}(A)$ if and only if for every μ_{ij}-semi open G containing x, $G \cap A \neq \phi$.

(d) $c_{\sigma_{ij}} \in \Gamma_{012+}$.

Proposition 1.5. (Fathima and Rani, 2019)Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following hold.

(a) $\left(i_{\sigma_{ij}}\right)^* = c_{\sigma_{ij}}$.

(b) $\left(c_{\sigma_{ij}}\right)^* = i_{\sigma_{ij}}$.

(c) $i_{\sigma_{ij}}(X-A) = X - c_{\sigma_{ij}}(A)$ for every subset A of X.

(d) $c_{\sigma_{ij}}(X-A) = X - i_{\sigma_{ij}}(A)$ for every subset A of X.

Proposition 1.6. (Fathima and Rani, 2019)Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following hold.

(a) A is μ_{ij}-semi open if and only if A is $c_{\mu_i} i_{\mu_j}$ - open if and only if $A = i_{c_{\mu_i} i_{\mu_j}}(A)$.

(b) $i_{\sigma_{ij}}(A) = i_{c_{\mu_i} i_{\mu_j}}$ and $c_{\sigma_{ij}} = c_{c_{\mu_i} i_{\mu_j}}$.

(c) $i_{\sigma_{ij}}(A) = A \cap c_{\mu_i} i_{\mu_j}(A)$.

(d) $c_{\sigma_{ij}}(A) = A \cap i_{\mu_i} c_{\mu_j}(A)$.

Similar results from (Jamuna Rani and Anees Fathima, 2021; Rani and Fathima, 2020; Jamuna Rani and Anees Fathima, 2020) are also used in the next section.

3 Relationship between the operators:

The following theorem gives some of the relationships between $i_{\sigma_{ij}}$, $c_{\sigma_{ij}}$, $i_{\alpha_{ij}}$, $c_{\alpha_{ij}}$, i_{μ_i} and c_{μ_i}.

Theorem 1.1.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $i_{\sigma_{ij}}(A) = A \cap c_{\mu_i} i_{\mu_j}(A)$

(b) $c_{\sigma_{ij}}(A) = A \cup i_{\mu_i} c_{\mu_j}(A)$

(c) $i_{\alpha_{ij}}(A) = A \cap i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$

(d) $c_{\alpha_{ij}}(A) = A \cup c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$

(e) $i_{\mu_i}(c_{\sigma_{ij}}(A)) = i_{\mu_i} c_{\mu_j}(A)$

(f) $c_{\mu_i}(i_{\sigma_{ij}}(A)) = c_{\mu_i} i_{\mu_j}(A)$

(g) $c_{\sigma_{ij}}(i_{\mu_i})(A) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$

(h) $i_{\sigma_{ij}}(c_{\mu_i})(A) = c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$

(i) $c_{\sigma_{ji}} i_{\sigma_{ij}}(A) = (A \cap c_{\mu_i} i_{\mu_j}(A)) \cup i_{\mu_j} c_{\mu_i} i_{\mu_j}(A)$ for every subset A of X.

(j) $i_{\sigma_{ij}}c_{\sigma_{ji}}(A) = (A \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)) \cup i_{\mu_j}c_{\mu_i}(A)$ for every subset A of X.

Proof. (a) By theorem 1.6(b), $i_{\sigma_{ij}} = i_{c_{\mu_i}i_{\mu_j}}$ implies $i_{\sigma_{ij}}(A) \subset A \cap c_{\mu_i}i_{\mu_j}(A)$. For the reverse part, $A \cap c_{\mu_i}i_{\mu_j} \subset c_{\mu_i}i_{\mu_j}(A) \subset c_{\mu_i}i_{\mu_j}\left(A \cap c_{\mu_i}i_{\mu_j}(A)\right)$. Thus $A \cap c_{\mu_i}i_{\mu_j} = i_{c_{\mu_i}i_{\mu_j}}(A \cap c_{\mu_i}i_{\mu_j}) \subset i_{\sigma_{ij}}(A)$.
(b) The result follows from (a).
(c) The proof is similar to the proof of (a).
(d) The result follows from (c).
(e) $i_{\mu_i}\left(c_{\sigma_{ij}}(A)\right) = i_{\mu_i}\left(A \cup i_{\mu_i}c_{\mu_j}(A)\right) \subset i_{\mu_i}\left(A \cup c_{\mu_j}(A)\right) = i_{\mu_i}c_{\mu_j}(A)$.
Also, by (b), $i_{\mu_i}c_{\sigma_{ij}}(A) \supset i_{\mu_i}i_{\mu_i}c_{\mu_j}(A) = i_{\mu_i}c_{\mu_j}(A)$.
(f) The result follows from (e).
(g) By (b), $c_{\sigma_{ij}}(i_{\mu_i}(A)) = i_{\mu_i}(A) \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(A) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.
(h) By(a), $i_{\sigma_{ij}}(c_{\mu_i}(A)) = c_{\mu_i}(A) \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A) = c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$.
(i) By(a),$c_{\sigma_{ji}}i_{\sigma_{ij}}(A) = (A \cap c_{\mu_i}i_{\mu_j}(A)) \cup i_{\mu_j}c_{\mu_i}(A \cap c_{\mu_i}i_{\mu_j}(A))$.
Here, $c_{\mu_i}(A \cap c_{\mu_i}i_{\mu_j}(A)) \subset c_{\mu_i}i_{\mu_j}(A)$ and $c_{\mu_i}(A \cap c_{\mu_i}i_{\mu_j}(A)) \supset c_{\mu_i}\left(i_{\mu_j}(A) \cap c_{\mu_i}i_{\mu_j}(A)\right)$ $= c_{\mu_i}i_{\mu_j}(A)$.
(j) Now, $i_{\sigma_{ij}}c_{\sigma_{ji}}(A) = (A \cup i_{\mu_j}c_{\mu_i}(A)) \cap c_{\mu_i}i_{\mu_j}(A \cup i_{\mu_j}c_{\mu_i}(A))$ $= (A \cup i_{\mu_j}c_{\mu_i}(A)) \cap c_{\mu_j}i_{\mu_j}c_{\sigma_{ji}}(A) = \left(A \cup i_{\mu_j}c_{\mu_i}(A)\right) \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$ by (e). Hence $i_{\sigma_{ij}}c_{\sigma_{ji}}(A) = \left(A \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)\right) \cup i_{\mu_j}c_{\mu_i}(A)$. □

The following theorem shows some results for the operators $i_{\pi_{ij}}, c_{\pi_{ij}}, i_{\beta_{ij}}, c_{\beta_{ij}}$.

Theorem 1.2.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $i_{\pi_{ij}}(A) = A \cap i_{\mu_i}c_{\mu_j}(A)$

(b) $c_{\pi_{ij}}(A) = A \cup c_{\mu_i}i_{\mu_j}(A)$

(c) $i_{\beta_{ij}}(A) = A \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$

(d) $c_{\beta_{ij}}(A) = A \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$

(e) $i_{\mu_i}(i_{\sigma_{ji}}(A)) = i_{\mu_i}(A)$

(f) $c_{\mu_i}(c_{\sigma_{ji}}(A)) = c_{\mu_i}(A)$.

Proof. (a) In (Rani and Fathima, 2020), by theorem 3.2(e), we have $i_{\pi_{ij}} = i_{i_{\mu_i}c_{\mu_j}}$ and so $i_{\pi_{ij}}(A) \subset A \cap i_{\mu_i}c_{\mu_j}(A)$. Let $x \in i_{\mu_i}c_{\mu_j}(A)$. Let G be any μ_i open set containing x such that $G \cap i_{\mu_i}c_{\mu_j}(A)$ is a μ_i -open set containing x. since $x \in c_{\mu_j}(A)$ and $G \cap i_{\mu_i}c_{\mu_j}(A) \cap A \neq \phi$ and so $x \in c_{\mu_j}\left(i_{\mu_i}c_{\mu_j}(A) \cap A\right)$. Therefore, $i_{\mu_i}c_{\mu_j}(A) \subset c_{\mu_j}i_{\mu_i}c_{\mu_j}(A) \cap A$ and so $i_{\mu_i}c_{\mu_j}(A) \subset i_{\mu_i}c_{\mu_j}(A \cap i_{\mu_i}c_{\mu_j}(A))$, $A \cap i_{\mu_i}c_{\mu_j}(A) \subset i_{\mu_i}c_{\mu_j}(A) \subset i_{\mu_i}c_{\mu_j}\left(A \cap i_{\mu_i}c_{\mu_j}(A)\right), A \cap i_{\mu_i}c_{\mu j}(A)$

$= i_{i_{\mu_i} c_{\mu_j}} \big(A \cap i_{\mu_i} c_{\mu_j}(A)\big) \subset i_{\pi_{ij}}(A) \cap i_{\mu_i} c_{\mu_j}(A) \subset i_{\pi_{ij}}(A)$. Hence $i_{\pi_{ij}}(A) = A \cap i_{\mu_i} c_{\mu_j}(A)$.
(b) The result follows from (a).
(c) The proof is similar to the proof of (a).
(d) By theorem 3.10(i) in (Jamuna Rani and Anees Fathima, 2021), the proof is similar to the proof of (b).
(e) $i_{\mu_i}(i_{\sigma_{ji}}(A)) = i_{\mu_i}(A \cap c_{\mu_j} i_{\mu_i}(A))$ by theorem 1.1(a) and so $i_{\mu_i}(i_{\sigma_{ji}}(A)) = i_{\mu_i}(A) \cap i_{\mu_i} c_{\mu_j} i_{\mu_i}(A) = i_{\mu_i}(A)$ by proposition 1.3(b).
(f) The result follows from (e). □

The following theorem gives characterizations of μ_{ij}-semi open, μ_{ij}-preopen, $\alpha\mu_{ij}$-open, $\beta\mu_{ij}$-open sets using the interior and closure operators.

Theorem 1.3.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $A \in \pi_{ij}(\mu)$ if and only if $c_{\sigma_{ij}}(\mu) = i_{\mu_i} c_{\mu_j}(A)$

(b) A is μ_{ij}-preclosed if and only if $i_{\sigma_{ij}}(A) = c_{\mu_i} i_{\mu_j}(A)$.

(c) $A \in \sigma_{ij}(\mu)$ if and only if $c_{\pi_{ij}}(A) = c_{\mu_i} i_{\mu_j}(A)$.

(d) A is μ_{ij}-semiclosed if and only if $i_{\pi_{ij}}(A) = i_{\mu_i} c_{\mu_j}(A)$.

(e) $A \in \alpha_{ij}(\mu)$if and only if $c_{\beta_{ij}}(A) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

(f) A is $\alpha\mu_{ij}$ -closed if and only if $i_{\beta_{ij}}(A) = c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

(g) $A \in \beta_{ij}(\mu)$ if and only if $c_{\alpha_{ij}}(A) = c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

(h) A is $\beta_{\mu_{ij}}$-closed if and only if $i_{\alpha_{ij}}(A) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

Proof. The proof follows from theorem 1.1(b),1.2(b),1.2(d), 1.1(d).□

Theorem 1.4.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $c_{\mu_i}(c_{\pi_{ij}}(A)) = c_{\mu_i}(A)$.

(b) $i_{\mu_i}(i_{\pi_{ij}}(A)) = i_{\mu_i}(A)$.

(c) $c_{\mu_i}(c_{\beta_{ji}}(A)) = c_{\mu_i}(A)$.

(d) $i_{\mu_i}(i_{\beta_{ji}}(A)) = i_{\mu_i}(A)$.

(e) $i_{\mu_i}(c_{\pi_{ji}}(A)) \subset c_{\mu_j}i_{\mu_i}(A)$ and $c_{\mu_j}i_{\mu_i}(c_{\pi_{ji}}(A)) = c_{\mu_j}i_{\mu_i}(A)$.

(f) $i_{\mu_i}(c_{\pi_{ji}}(A)) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.

(g) $c_{\beta_{ij}}(i_{\mu_i}(A)) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A) = i_{\mu_i}(c_{\beta_{ij}}(A))$.

(h) $c_{\beta_{ij}}(i_{\beta_{ji}}(A)) = i_{\beta_{ji}}(c_{\beta_{ij}}(A)) = \left(A \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)\right) \cap c_{\mu_j}i_{\mu_i}c_{\mu_j}(A)$.

(i) $i_{\beta_{ij}}(c_{\mu_i}(A)) = c_{\mu_i}i_{\mu_j}c_{\mu_i}(A) = c_{\mu_i}i_{\beta_{ij}}(A)$.

(j) $c_{\sigma_{ij}}\left(i_{\sigma_{ji}}(A)\right) \subset i_{\beta_{ji}}\left(c_{\beta_{ij}}(A)\right) \subset i_{\sigma_{ji}}\left(c_{\sigma_{ij}}(A)\right)$.

Proof. (a) Clearly, $c_{\mu_i}(A) \subset c_{\mu_i}\left(c_{\pi_{ij}}(A)\right)$.
Again, $c_{\mu_i}\left(c_{\pi_{ij}}(A)\right) \subset c_{\mu_i}(c_{\mu_i}(A)) = c_{\mu_i}(A)$.
(b) The proof follows from (a).
The proof of (c) and (d) are similar to (a) and (b).
(e) Similar proof, so omitted.
(f) By (e), $i_{\mu_i}\left(c_{\pi_{ji}}(A)\right) \subset i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$. Again, $i_{\mu_i}\left(c_{\pi_{ji}}(A)\right) = i_{\mu_i}\left(A \cup c_{\mu_j}i_{\mu_i}(A)\right) \supset i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$. Therefore, $i_{\mu_i}\left(c_{\pi_{ji}}(A)\right) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.
(g) By theorem 1.2(d), $c_{\beta_{ij}}(i_{\mu_i}(A)) = i_{\mu_i}(A) \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(i_{\mu_i}(A)) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.
Again, $i_{\mu_i}\left(c_{\beta_{ij}}(A)\right) = i_{\mu_i}\left(A \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)\right) \supset i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.
For Converse part, $i_{\mu_i}\left(c_{\beta_{ij}}(A)\right) \subset i_{\mu_i}\left(c_{\pi_{ij}}(A)\right) \subset c_{\mu_j}i_{\mu_i}(A) \subset i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.
(h) Similar proof, so omitted.
(i) The proof follows from (g).
(j) The proof follows from 1.1(i) and 1.1(j).□

The following theorem gives a characterization of $\beta_{ij}(\mu)$-open sets.

Theorem 1.5.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following are equivalent.

(a) $A \in \beta_{ji}(\mu)$.

(b) $A \subset i_{\beta_{ji}}\left(c_{\beta_{ij}}(A)\right)$.

(c) $A \subset i_{\sigma_{ji}}\left(c_{\sigma_{ij}}(A)\right)$.

Proof. $(a) \Rightarrow (b)$ If $A \in \beta_{ji}(\mu)$, then $A = i_{\beta_{ji}}(A) \subset i_{\beta_{ji}}\left(c_{\beta_{ij}}(A)\right)$.
$(b) \Rightarrow (c)$ By theorem 1.4(j), the proof follows.
$(c) \Rightarrow (a)$ $A \subset i_{\sigma_{ji}}\left(c_{\sigma_{ij}}(A)\right) \Rightarrow A \subset \left(c_{\sigma_{ij}}(A) \cap c_{\mu_j} i_{\mu_i}\left(c_{\sigma_{ij}}(A)\right)\right) = \left(c_{\sigma_{ij}}(A) \cap c_{\mu_j} i_{\mu_i} c_{\mu_j}(A)\right)$ by theorem 1.1(e) and so $A \subset c_{\mu_j} i_{\mu_i} c_{\mu_j}(A)$. □

In the following theorem, we prove some relationships between $i_{\alpha_{ij}}$,$c_{\alpha_{ij}}$ with i_{μ_i}, c_{μ_i}.

Theorem 1.6.
Let (X, μ_1, μ_2) be a bigenralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $c_{\mu_i} c_{\alpha_{ij}}(A) = c_{\alpha_{ij}} c_{\mu_i}(A) = c_{\mu_i}(A)$.

(b) $i_{\mu_i} i_{\alpha_{ij}}(A) = i_{\alpha_{ij}} i_{\mu_i}(A) = i_{\mu_i}(A)$.

(c) $c_{\alpha_{ij}}\left(i_{\mu_j}(A)\right) = c_{\mu_i} i_{\mu_j}(A)$.

d) $c_{\mu_j}\left(i_{\alpha_{ij}}(A)\right) = c_{\mu_j} i_{\mu_i}(A)$.

(e) $i_{\alpha_{ij}}\left(c_{\mu_j}(A)\right) = i_{\mu_i} c_{\mu_j}(A)$.

(f) $i_{\mu_j}\left(c_{\alpha_{ij}}(A)\right) = i_{\mu_j} c_{\mu_i}(A)$.

Proof. (a) The proof follows from the theorem 1.1(d).
(b) The proof of (b) follows from (a).
(c) The proof follows from the theorem 1.1(d).
(d) $c_{\mu_j}\left(i_{\alpha_{ij}}(A)\right) \subset c_{\mu_j}\left(c_{\mu_j}(A) \cap i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)\right) = c_{\mu_j} i_{\mu_i}(A)$.
Again, $c_{\mu_j}\left(i_{\alpha_{ij}}(A)\right) \supset c_{\mu_j}\left(A \cap i_{\mu_i}(A)\right) = c_{\mu_j} i_{\mu_i}(A)$.
(e) The proof of (e) follows from (c).
(f) The proof of (f) follows from (d). □

The following theorem shows some relationships between $i_{\alpha_{ij}}$, $c_{\alpha_{ij}}$ with $i_{\sigma_{ij}}$, $c_{\sigma_{ij}}$, $i_{\pi_{ij}}$,$c_{\pi_{ij}}$, $i_{\beta_{ij}}$ and $c_{\beta_{ij}}$.

Theorem 1.7.
Let (X, μ_1, μ_2) be a bigenralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $i_{\alpha_{ij}}\left(c_{\sigma_{ij}}(A)\right) = i_{\mu_i} c_{\mu_j}(A)$.

(b) $i_{\alpha_{ij}}\left(c_{\pi_{ji}}(A)\right) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

(c) $i_{\alpha_{ij}}\left(c_{\beta_{ji}}(A)\right) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

(d) $c_{\alpha_{ij}}\left(i_{\sigma_{ij}}(A)\right)=c_{\mu_i}i_{\mu_j}(A)$.

(e) $c_{\alpha_{ij}}\left(i_{\pi_{ji}}(A)\right) = c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$.

(f) $c_{\alpha_{ij}}\left(i_{\beta_{ji}}(A)\right) = c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$.

Proof. (a) The proof follows from the theorem 1.1(c), 1.1(f).
(b) The proof follows from the theorem 1.1(c), 1.4(g) and 1.1(a).
(c) The proof follows from the theorem 1.1(c), 1.4(h).
(d) The proof of (d) follows from (a).
(e) The proof of (e) follows from (b).
(f) The proof of (f) follows from (c). □

Theorem 1.8.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $i_{\sigma_{ij}}c_{\alpha_{ij}}(A) = c_{\alpha_{ij}}(A) \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$

(b) $c_{\sigma_{ij}}i_{\alpha_{ij}}(A) = i_{\alpha_{ij}}(A) \cap i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$

(c) $c_{\sigma_{ij}}c_{\alpha_{ji}}(A) =c_{\alpha_{ji}}(A) \cap c_{\sigma_{ij}}(A)$.

(d) $i_{\sigma_{ij}}i_{\alpha_{ji}}(A) =i_{\alpha_{ji}}(A) \cap i_{\sigma_{ij}}(A)$.

(e) $i_{\sigma_{ij}}i_{\beta_{ij}}(A) = i_{\sigma_{ij}}(A)$.

(f) $c_{\sigma_{ij}}c_{\beta_{ij}}(A) = c_{\sigma_{ij}}(A)$.

Proof. (a) The result follows from the theorem 1.1(a) and 1.6(f).
(b) The proof of (b) follows from (a).
(c) The result follows from the theorem 1.1(b) and 1.6(a).
(d) The proof of (d) follows from (c).
(e) The result follows from the theorem 1.1(a), 1.4(d) and 1.2(c).
(f) The proof of (f) follows from (e).

Theorem 1.9.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $i_{\pi_{ij}}c_{\alpha_{ji}}(A) = c_{\alpha_{ji}}(A) \cap i_{\mu_i}c_{\mu_j}(A)$.

(b) $i_{\pi_{ij}}i_{\beta_{ji}}(A) = i_{\pi_{ij}}(A)$.

(c) $c_{\pi_{ij}} i_{\alpha_{ji}}(A) = i_{\alpha_{ji}} \cap c_{\mu_i} i_{\mu_j}(A)$.

(d) $c_{\pi_{ij}} c_{\beta_{ji}}(A) = c_{\pi_{ij}}(A)$.

Proof. (a) The result follows from the theorem 1.2(a) and 1.6(a).
(b) The result follows from the theorem 1.2(a) and 1.4(i).
(c) The proof of (c) follows from (a).
(d) The proof of (d) follows from (b).□

Theorem 1.10.
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X and $\mu_i \in \Gamma_4$. Then the following hold.

(a) $i_{\beta_{ij}} c_{\alpha_{ij}}(A) = c_{\alpha_{ij}}(A) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$= $c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

(b) $c_{\beta_{ij}} i_{\alpha_{ij}}(A) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(A) = i_{\alpha_{ij}}(A) \cap i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

Proof. (a) The result follows from the theorem 1.2(c) and 1.6(a).
(b) The proof of (b) follows from (a). □

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