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The Edge-To-Vertex Triangle Free Detor Distance in Graphs

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Abstract

For every connected graph G, the triangle free detour distance $D_{\Delta f}$ (u, v) is the length of a longest u- v triangle free path in G, where u, v are the vertices of G. A u-v triangle free path of length $D_{\Delta f}$ (u, v) is called the u-v triangle free detour. In this article, the edge-to-vertex triangle free detour distance is introduced. It is found that the edge -to-vertex triangle free detour distance differs from the edge -to-vertex distance and edge-to-vertex detour distance. The edge-to-vertex triangle free detour distance is found for some standard graphs. Their bounds are determined and their sharpness is checked. Certain general properties satisfied by them are studied.

Keywords: connected graph, edge -to-vertex distance and edge-to-vertex detour distance

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1. Introduction

The facility location problem was introduced as edge-to-vertex distance by Santhakumaran [9], in 2010. For an edge e and a vertex v in a connected graph, the edge-to-vertex distance is defined by $d(e,v) = min\{d(u,v) : u \in e\}$. The edge-to-vertex eccentricity of e is defined by $e_2(e) = max\{d(e,v) : v \in V\}$. A vertex v of G such that $e_2(e) = d(e,v)$ is called an edge-to-vertex eccentric vertex of v. The edge-to-vertex radius r_2 of G is defined by $r_2 = min\{e_2(e) : e \in E\}$ and the edge-to-vertex diameter d_2 of G is defined by$d_2 = max\{e_2(e) : e \in E\}$. An edge e for which $e_2(e)$ is minimum is called an edge-to-vertex central edge of $G$and the set of all edge-tovertex central edges of G is the edge-to-vertex center $C_2(G)$ of$G$. An edge e for which $e_2(e)$ is maximum is called an edge-to-vertexperipheral edge of G and the set of all edge-to-vertex peripheraledges of G is the edge-to-vertex periphery $P_2(G)$ of G. If every edgeof $G$is an edge-to-vertex central edge then G is called the edge-to-vertex self-centered graph. This concept is useful in channel assignment problem in radio technology and security-based communication network design. The concept of edge-to-vertex detour distance was introduced by I. Keerthi Asir [6], Let e be an edge and v a vertex in a connected graph G. An edge-to-vertex $e-v$ path P is a $u-v$ path, where$u$ is a vertex in e such that P contains no vertices of e other than u. The edge-to-vertex detour distance $D(e,v)$ is the length of a longest $e-v$path in G. An$e-v$ path of length $D(e,v)$ is called an edge-to-vertex $e-v$ detour or simply $e-v$ detour. Forour convenience an$e-v$ path of length $d(e,v)$ is called an edge-to-vertex $e-v$ geodesic or simply $e–v$ geodesic.

The following theorems are used in the article.

Theorem: 1.1.[6] For any edge e and a vertex $v$in a non-trivial connected graph of order$n$, $0 \leq d(e,v) \leq D(e,v) \leq n-2$.

Theorem: 1.2.[6] Let$K_{n,m}$ $(n < m)$ be a complete bipartite graph with partitionV_1, V_2 of $V(K_{n,m})$ such that $|V_1| = n$ and $|V_2| = m$. Let e be an edge and v a vertex such that $v \notin e$ in $K_{n,m}$, then

$$D(e,v) = \begin{cases} 2n-2 & if v \in V_1 \\ 2n-1 & if v \in V_2 \end{cases}$$

2. Edge-To-Vertex Triangle Free Detour Distance

Definition. 2.1 Let Gbe a connected graph. Let $e$be an edge and u a vertex in$G$. An edge-to-vertex $e-u$triangle free path $P$is a $u-v$ triangle free path, where$v$is a vertex in $e$such that Pcontains no vertices of e other than v. The edge-to-vertex triangle free detour distance is the length of the longest $e-u$ triangle free path in G.It is denoted by $D_{\Delta f}(e,v)$. An$e-u$ triangle free path of length$D_{\Delta f}(e,v)$is called an edge-to-vertex $e-u$triangle free detour.

Example: 2.1 Consider the graph Ggiven in the figure: 2.1. Let $e = \{u_6, u_7\}$and $v = u_4$. The paths between $e$and v are P_1: u_6, u_5, u_4;P_2: u_7, u_2, u_4;P_3: u_7, u_2, u_3, u_4 ;P_4: $u_7, u_8, u_9, u_1, u_2, u_4$; and P_5: $u_7, u_8, u_9, u_1, u_2, u_3, u_4$;The paths $P_1, P_2, P_4$are triangle free $e - v$ paths and P_3 and P_5 are not triangle free $e - v$ paths. Thus edge-to-vertex distance $d(e, v) = 2$, edge-to-vertex triangle free detour distance $D_{\Delta f}(e, v) = 5$ and edge-to-vertex detour distance $D(e, v) = 6$.

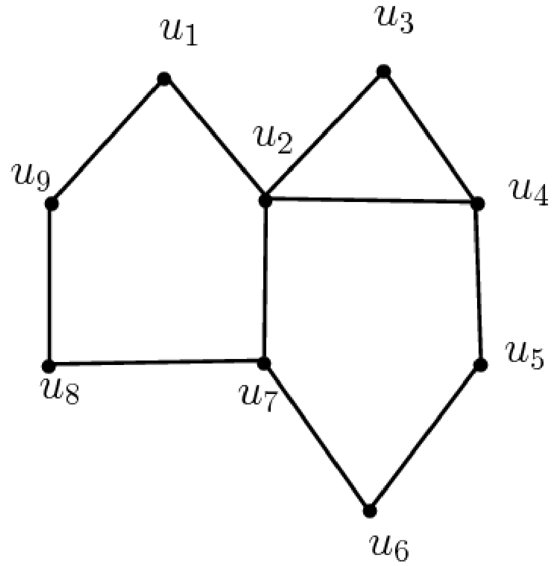

Figure: 2.1 G

Thus edge-to-vertex triangle free detour distance differs from the edge-to-vertex distance and edge-to-vertex detour distance.

Theorem. 2.1 Let G be a connected graph of order n. Let e be an edge and ua vertex of$G$, then $0 \leq d(e, v) \leq D_{\Delta f}(e, v) \leq D(e, v) \leq n - 2$.

Proof. By theorem 1.1 , we can conclude that $0 \leq d(e, v) \leq D(e, v) \leq n - 2$. It is enough to prove that (i)$d(e, v) \leq D_{\Delta f}(e, v)$ and (ii) $D_{\Delta f}(e, v) \leq D(e, v)$.

Thus (i) is true by the definition of edge-to-vertex distance and edge-to-vertex triangle free detour distance.

To prove :(ii)

Case(i): If the detour path does not induce a triangle in G, then$D_{\Delta f}(e, v) = D(e, v)$.

Case(ii): If the detour path induces a triangle in G, then $D_{\Delta f}(e, v) < D(e, v)$

Remark 2.1. The bounds in the theorem 2.1 are sharp. Let Gbe a graph and e be an edge, $d(e, u) = D_{\Delta f}(e, u) = D(e, u) = 0iffu \in e$.Let Gbe a path with vertices $\{v_1, v_2, \ldots . v_n\}$. Then$d(e, u) = D_{\Delta f}(e, u) = D(e, u) = n - 2$, where $e =$

$\{v_{n-1}, v_n\}$and $u = v_1$. Let Gbe a tree, $d(e,u) = D_{\Delta f}(e,u) = D(e,u)$ for every edge $e$and vertex $u$of G. For the graph G given in the figure:2.1, $e = \{u_6, u_7\}$ and $v = u_4$. The paths between $e$and v are $P_1: u_6, u_5, u_4$; $P_2: u_7, u_2, u_4$; $P_3: u_7, u_2, u_3, u_4$; $P_4: u_7, u_8, u_9, u_1, u_2, u_4$; and $P_5: u_7, u_8, u_9, u_1, u_2, u_3, u_4$;The paths $P_1, P_2, P_4$are triangle free $e - v$ paths and P_3 and P_5 are not triangle free $e - v$ paths. Thus edge-to-vertex distance $d(e,v) = 2$, edge-to-vertex triangle free detour distance $D_{\Delta f}(e,v) = 5$ and edge-to-vertex detour distance$D(e,v) = 6$. Thus $0 < d(e,v) < D_{\Delta f}(e,v) < D(e,v) < n-2$.

Theorem. 2.2 For a complete bipartite graph $G$with partitions $V_1$and V_2such that $|V_1| = n$and $|V_2| = m (n < m)$.Let e be an edge of G and ua vertex such that $u \notin e$ in G. Then, $D_{\Delta f}(e,u) = \begin{cases} 2n-2 & if u \in V_1 \\ 2n-1 & if u \in V_2 \end{cases}$

Proof. Since any vertex subset of G do not induce a cycle C_3in G. Thus edge-to-vertex triangle free detour distance is equal to edge-to-vertex detour distance. By theorem: 1.2, $D_{\Delta f}(e,u) = \begin{cases} 2n-2 & if u \in V_1 \\ 2n-1 & if u \in V_2 \end{cases}$

Corollary:2.1 Let G be a complete bipartite graph $K_{n,n}$ with partitions $V_1$and V_2 .Let e be an edge and u be a vertex such that $u \notin e$in$G$. Then$D_{\Delta f}(e,u) = 2n-2$.

Theorem: 2.3 Let G be a tree, then for every edge e and a vertex $v$in G, $d(e,v) = D_{\Delta f}(e,v) = D(e,v)$.

Remark: 2.2 The converse of the theorem:2.3 need not be true. Consider the graph,$G = C_4$, where $d(e,v) = D_{\Delta f}(e,v) = D(e,v) = 1$ if $v \notin e$ and $d(e,v) = D_{\Delta f}(e,v) = D(e,v) = 0$ if $v \in e$.

Definition: 2.2 The edge-to-vertex triangle free detour eccentricity $e_{\Delta f2}(e)$ of an edge e in a connected graph G is defined as$e_{\Delta f2}(e) = max\{D_{\Delta f}(e,v) : v \in V\}$. A vertex v for which $e_{\Delta f2}(e) = D_{\Delta f}(e,v)$ is called an edge-to-vertex triangle free detour eccentric vertex of e. The edge-to-vertex triangle free detour radius of G is defined as $R_{\Delta f2} = rad_{\Delta f2}(G) = min\{e_{\Delta f2}(e): e \in E\}$. The edge-to-vertex triangle free detour diameter of G is defined as $D_{\Delta f2} = diam_{\Delta f2}(G) = max\{e_{\Delta f2}(e): e \in E\}$.

Definition: 2.3 An edge e is called an edge-to-vertex triangle free detour central edge if $e_{\Delta f2}(e) = R_{\Delta f2}$. The edge-to-vertex triangle free detour center of $G$is defined as $C_{\Delta f2}(G) = Cen_{\Delta f2}(G) = \{e \in E: e_{\Delta f2}(e) = R_{\Delta f2}\}$.

Definition: 2.4 An edge e is called an edge-to-vertex triangle free detour peripheral edge if $e_{\Delta f2}(e) = D_{\Delta f2}$. The edge-to-vertex triangle free detour periphery of G is defined as $P_{\Delta f2}(G) = Per_{\Delta f2}(G) = \{e \in E: e_{\Delta f2}(e) = D_{\Delta f2}\}$.

Definition. 2.5 If every edge of a graph G is a edge-to-vertex triangle free detour central edge, then G is called edge-to-vertex triangle free detour self centered graph.

Definition. 2.6 If G is the edge-to-vertex triangle free detour self centered graph, then$G$ is called edge-to-vertex triangle free detour periphery.

Example. 2.2 For the graph Ggiven in the figure: 2.2, $e_1 = \{u_1, u_2\}$, $e_2 = \{u_2, u_3\}, e_3 = \{u_3, u_4\}, e_4 = \{u_4, u_5\}, e_5 = \{u_5, u_6\}, e_6 = \{u_6, u_7\}, e_7 = \{u_7, u_8\}, e_8 = \{u_1, u_8\}, e_9 = \{u_8, u_2\}, e_{10} = \{u_7, u_5\}, e_{11} = \{u_5, u_2\}, e_{12} = \{u_3, u_5\}$ are the edges of G.

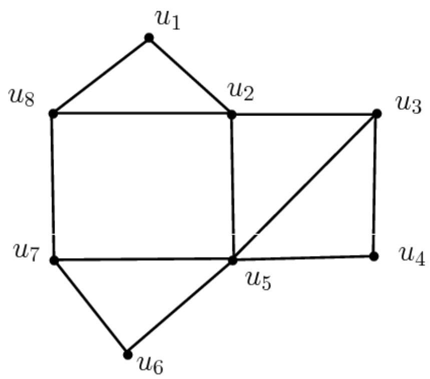

Figure:2.2 G

The edge-to- vertex triangle free detour distances of the graph G, are provided in the following table.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	$e_{\Delta f2}$
e_1	0	0	4	4	3	3	2	3	4
e_2	1	0	0	4	3	3	2	3	4
e_3	4	4	0	0	1	4	3	3	4
e_4	3	3	1	0	0	5	4	3	5
e_5	3	3	4	5	0	0	3	2	5
e_6	3	2	3	4	3	0	0	3	4
e_7	3	2	2	3	2	3	0	0	3
e_8	0	3	3	3	2	3	3	0	3
e_9	1	0	3	3	2	2	2	0	3
e_{10}	2	2	3	4	0	1	0	2	4
e_{11}	3	0	2	2	0	3	2	2	3
e_{12}	3	3	0	1	0	4	3	2	4

Table:2.1

The following table provides the edge-to- vertex distances, edge-to-vertex triangle free detour distances and edge-to- vertex detour distances of the graph Gin figure:2.2

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
e_2	2	2	2	2	2	2	2	3	2	2	1	2
$e_{\Delta f2}$	4	4	4	5	5	4	3	3	3	4	3	4
e_{D2}	6	6	6	6	6	6	6	6	5	5	4	5

Table: 2.2

The edge-to-vertex radius$r_2 = 1$, the edge-to-vertex triangle free detour radius $R_{\Delta f2} = 3$, the edge-to- vertex detour radius $R_2 = 4$. Thus, the edge-to-vertex triangle free detour radius is different from the edge-to- vertex radius and the edge-to- vertex detour radius. The edge-to-vertex diameter $d_2 = 3$, the edge-to- vertex triangle free detour diameter $D_{\Delta f2} = 6$, the edge-to- vertex detour diameter $D_2 = 6$. Thus, the edge-to- vertex triangle free detour diameter is different from the edge-to- vertex diameter and the edge-to- vertex detour diameter.

The edge-to-vertex center $C_2(G) = \{e_{11}\}$, the edge-to-vertex triangle free detour center $C_{\Delta f2}(G) = \{e_7,\ e_8,\ e_9,\ e_{11}\}$, the edge-to-vertex detour center $C_{D2}(G) = \{e_9,\ e_{10},\ e_{11}\}$Thus the edge-to- vertex triangle free detour center is different from the edge-to- vertex center and the edge-to- vertex detour center. The edge-to-vertex periphery $P_2(G) = \{e_8\}$, the edge-to-vertex triangle free detour periphery $P_{\Delta f2}(G) = \{e_4,\ e_5\}$, the edge-to-vertex detour periphery $P_{D2}(G) = \{e_1, e_2, e_3, e_4,\ e_5, e_6,\ e_7, e_8\}$. Thus, the edge-to- vertex triangle free detour periphery is different from the edge-to-vertex periphery and the edge-to- vertex detour periphery.

The edge-to-vertex triangle free detour radius $R_{\Delta f2}$ and the edge-to-vertex triangle free detour diameter $D_{\Delta f2}$ of some standard graphs are provided in the table:2.3

G	K_n	P_n	$C_n\ (n \geq 4)$	$W_n (n \geq 5)$	$K_{n,m} (n \geq m)$
$R_{\Delta f2}$	1	$\left\lfloor \frac{n-2}{n} \right\rfloor$	$n-2$	$n-2$	$\begin{cases} 2(n-1), if n = m \\ 2n-1,\ \ if n > m \end{cases}$
$D_{\Delta f2}$	1	$n-2$	$n-2$	$n-2$	$\begin{cases} 2(n-1), if n = m \\ 2n-1,\ \ if n > m \end{cases}$

Example: 2.3 The complete graph K_n, the Cycle graph $C_n\ (n \geq 4)$ and the wheel graph $W_n(n \geq 5)$ are the edge-to-vertex triangle free detour self centered graph.

Theorem:2.4 For a connected graph G of order n. Then
(i)$0 \leq e_2(e) \leq e_{\Delta f2}(e) \leq e_{D2}(e) \leq n-2$, for every edge $e$of G.
(ii)$0 \leq r_2 \leq R_{\Delta f2} \leq R_2 \leq n-2$.
(iii) $0 \leq d_2 \leq D_{\Delta f2} \leq D_2 \leq n-2$.

Remark: 2.3 The bounds in the theorem:2.4are sharp. If $G = P_2$, then $e_2(e) = e_{\Delta f2}(e) = e_{D2}(e) = 0$. If $G = C_n\ (n \geq 4)$, then $e_2(e) = e_{\Delta f2}(e) = e_{D2}(e) = n-2$. For the graph G given in the figure:2.2, $0 < e_2(e) < e_{\Delta f2}(e) < e_{D2}(e) < n-2$, for the edges $e = e_9, e_{10}, e_{11}, e_{12}$.

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