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$G_{\alpha,\beta}$ Antagonistic Intuitionistic Fuzzy Sub Commutative Ideals of Subtraction G-Algebra

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Abstract

The notions of $G_{\alpha,\beta}$ over antagonistic intuitionistic fuzzy sub commutative ideals of subtraction G-algebras are introduced. The characterization properties of antagonistic intuitionistic fuzzy sub commutative ideals are obtained. We investigate the relations between antagonistic intuitionistic fuzzy sub implicative ideals and antagonistic intuitionistic fuzzy sub commutative ideals of subtraction G-algebra.

Keywords: Subtraction G-algebra, AIFSI ideals, AIFSC ideals.

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1. Introduction

The introduction of fuzzy sets by Zadeh [17], there have been a number of generalizations of this fundamental concept. BCK-algebras and BCI-algebras are two classes of logical algebra. Extensive applications of fuzzy set theory have been found in various fields, for example, artificial intelligence, computer science, control engineering, expert system, management science, operation research and many others which were initiated by K. Iseki [3, 4]. The notion of fuzzy sets, invented by L. A. Zadeh [17], has been applied to many fields. Since then, fuzzy BCI/BCK algebras have been extensively investigated by several researchers. For BCK-algebras, Y. B. Jun et al. [7, 10] introduced the notions of fuzzy positive implicative ideals. Bandaru and Rafi [1] introduced a new notion called G-algebra, since the fuzzy G-algebras have been extensively investigated by several researches. The properties of fuzzy sub commutative ideals and fuzzy sub implicative ideals are obtained. Ragavan and Solairaj [14] some new results on intuitionistic fuzzy H-ideals in BCI algebra.

2. Preliminaries

Definition 2.1. [8] A Subtraction G-algebra we mean a nonempty set X with a binary operation - and a constant 0 satisfying the following conditions:
(F1) $x - x = 0$,
(F2) $x - (x - y) = y$, for all $x, y \in X$.

Definition 2.2. [15] (G-subalgebra) A non-empty subset S of a subtraction G-algebra X is called a subtraction G-subalgebra of X if $x-y \in S$ [0,1].

Definition 2.3. [16] A fuzzy set f of a universe X is a function from X to the unit closed interval [0, 1], that is $f: X \to [0, 1]$.

Definition 2.4. [2] An intuitionistic fuzzy set A in a finite universe of discourse $X = \{x_1, x_2, x_3, x_4, \cdots x_n\}$ is given by $A = \{<x, \Psi_A(x), \Omega_A(x)>: x \in X\}$, Where $\Psi_A: X \to [0, 1]$ and $\Omega_A : X \to [0, 1]$ such that $0 \leq \Psi_A(x) + \Omega_A(x) \leq 1$. The number $\Psi_A(x)$ and $\Omega_A(x)$ denote the degree of membership and non-membership of $x \in X$ to A, respectively. For each IFS A in X, if $\pi_A(x) = 1 - \Psi_A(x) - \Omega_A(x)$,for all $x \in X$.

Definition 2.5. A nonempty subset I of X is called an ideal of X if (I_1): $0 \in I$, and (I_2): $x - y \in I$ and $y \in I$ imply $x \in I$.

Definition 2.6. A fuzzy subset Ψ_A of X is said to be a fuzzy ideal of X if it satisfies
(F_3) $\Psi_A(0) \geq \Psi_A(x)$ for all $x \in X$,
(F_4) $\Psi_A(x) \geq \min\{\Psi_A(x - y), \Psi_A(y)\}$ for all $x, y \in X$.

Definition 2.7. An intuitionistic fuzzy set $A = \{x, \Psi_A(x), \Omega_A(x): x \in X\}$ in X is called an intuitionistic fuzzy ideal of X if it satisfies
(F_5) $\Psi_A(0) \geq \Psi_A(x)$, $\Omega_A(0) \leq \Omega_A(x)$,
(F_6) $\Psi_A(x) \geq \min\{\Psi_A(x - y), \Psi_A(y)\}$ and
(F_7) $\Omega_A(x) \leq \max\{\Omega_A(x - y), \Omega_A(y)\}$ for all $x, y \in X$.

Definition 2.8. An intuitionistic fuzzy set $A = \{x, \Psi_A(x), \Omega_A(x): x \in X\}$ in X is called an antagonistic -intuitionistic fuzzy ideal of X if it satisfies
(F_8) $\Psi_A(0) \leq \Psi_A(x)$, $\Omega_A(0) \geq \Omega_A(x)$,
(F_9) $\Psi_A(x) \leq \max\{\Psi_A(x - y), \Psi_A(y)\}$ and
(F_{10}) $\Omega_A(x) \geq \min\{\Omega_A(x - y), \Omega_A(y)\}$ for all $x, y \in X$.

Definition 2.9. [9, 11] A nonempty subset I of X is called a positive implicative ideal (i.e., weakly positive implicative ideal) of X if it satisfies (I_1) $0 \in I$ and (I_3): $((x - z) - z) - (y - z) \in I$ and $y \in I$ imply $x - z \in I$.

Definition 2.10. [9, 14] A nonempty subset I of X is called a sub-implicative ideal of X if it satisfies (I_1) $0 \in I$ and (I_3): $((x - (x - y)) - (y - x)) - z \in I$ and $z \in I$ imply $y - (y - x) \in I$.

Definition 2.11. [9, 13] A nonempty subset I of X is called a sub-commutative ideal of X if it satisfies (I_1) $0 \in I$ and (I_4): $(y - (y - (x - (x - y)))) - z \in I$ and $z \in I$ imply $x - (x - y) \in I$.

Definition 2.12. An anti-Fuzzy Subset Ψ_A of X is called an antagonistic -fuzzy sub-implicative ideal of X if it satisfies
(F_{11}) $\Psi_A(0) \leq \Psi_A(x)$ for all $x \in X$, and
(F_{12}) $\Psi_A(y - (y - x)) \leq \max\{\Psi_A(((x - (x - y)) - (y - x)) - z), \Psi_A(z)\}$ for all $x, y, z \in X$.

Definition 2.13. An anti-fuzzy subset Ψ_A of X is called an antagonistic -fuzzy sub-commutative ideal (briefly, AFSC-ideals) of X if it satisfies (F_1) and
(F_{13}) $\Psi_A(x - (x - y)) \leq \max\{\Psi_A((y - (y - (x - (x - y)))) - z), \Psi_A(z)\}$ for all $x, y, z \in X$.

Definition 2.14. An anti-Fuzzy Subset (Ψ_A, Ω_A) of X is called an antagonistic -intuitionistic fuzzy sub-implicative ideal of X if it satisfies
(F_{16}) $\Psi_A(0) \leq \Psi_A(x)$, $\Omega_A(0) \geq \Omega_A(x)$ for all $x \in X$, and
(F_{17}) $\Psi_A(y - (y - x)) \leq \max\{\Psi_A(((x - (x - y)) - (y - x)) - z), \Psi_A(z)\}$ for all $x, y, z \in X$.
(F_{18}) $\Omega_A(y - (y - x)) \geq \min\{\Omega_A(((x - (x - y)) - (y - x)) - z), \Omega_A(z)\}$ for all $x, y, z \in X$.

Definition 2.15. An anti-fuzzy subset (Ψ_A, Ω_A) of X is called an antagonistic intuitionistic fuzzy sub-commutative ideal (briefly, AFSC-ideals) of X if it satisfies
(F_{19}) $\Psi_A(0) \leq \Psi_A(x)$, $\Omega_A(0) \geq \Omega_A(x)$ for all $x \in X$, and
(F_{20}) $\Psi_A(x - (x - y)) \leq \max\{\Psi_A((y - (y - (x - (x - y)))) - z), \Psi_A(z)\}$ for all $x, y, z \in X$.

(F_{21}) $\Omega_A (x - (x - y)) \geq \min \{\Omega_A ((y - (y - (x - (x - y)))) - z), \Omega_A(z))\}$ for all x, y, z ∈ X.

3. $G_{\alpha,\beta}$ Over Antagonistic-Intuitionistic Fuzzy Sub Commutative Ideal of subtraction G-algebras

Theorem 3.1. If $G_{\alpha,\beta}(A)$ and $G_{\alpha,\beta}(B)$ are antagonistic -intuitionistic fuzzy sub-commutative ideal of subtraction G-algebra then $G_{\alpha,\beta}(A+B)$ is also antagonistic -intuitionistic fuzzy sub-commutative ideal of subtraction G-algebra.

Proof: Given $G_{\alpha,\beta}(A)$ and $G_{\alpha,\beta}(B)$ are antagonistic -intuitionistic fuzzy sub-commutative ideal of X.

(1) $\Psi_A (0) \leq \Psi_A(x)$, $\Omega_A (0) \geq \Omega_A (x)$ for all x ∈ X, and

(2) $\Psi_A (x - (x - y)) \leq \max \{\Psi_A ((y - (y - (x - (x - y)))) - z), \Psi_A (z)\}$ for all x, y, z ∈ X.

(3) $\Omega_A (x - (x - y)) \geq \min \{ \Omega_A ((y - (y - (x - (x - y)))) - z), \Omega_A (z))\}$ for all x, y, z ∈ X.

(4) $\Psi_B (0) \leq \Psi_B (x)$, $\Omega_B (0) \geq \Omega_B (x)$ for all x ∈ X, and

(5) $\Psi_B (x - (x - y)) \leq \max \{\Psi_B ((y - (y - (x - (x - y)))) - z), \Psi_B (z)\}$ for all x, y, z ∈ X.

(6) $\Omega_B (x - (x - y)) \geq \min \{\Omega_B ((y - (y - (x - (x - y)))) - z), \Omega_B (z))\}$ for all x, y, z ∈ X.

Case1:

$\Psi_A (0) + \Psi_B (0) - \Psi_A (0). \Psi_B (0) \leq \Psi_A (x) + \Psi_B (x) - \Psi_A (x). \Psi_B (x)$

$G_{\alpha,\beta} \{\Psi_A (0) + \Psi_B (0) - \Psi_A (0). \Psi_B (0)\} \leq G_{\alpha,\beta} \{\Psi_A (x) + \Psi_B (x) - \Psi_A (x). \Psi_B (x)\}$

$\{\alpha\Psi_A (0) + \alpha\Psi_B (0) - \alpha\Psi_A (0). \alpha \Psi_B (0)\} \leq \{\alpha \Psi_A(x) + \alpha \Psi_B (x) - \alpha \Psi_A (x). \alpha \Psi_B (x)\}$

$\{\alpha\Psi_A (0) + \alpha\Psi_B (0) – \alpha^2\Psi_A (0). \Psi_B (0)\} \leq \{\alpha\Psi_A (x)+ \alpha\Psi_B (x)- \alpha^2\Psi_A (x). \Psi_B (x)\}$

$G_{\alpha,\beta} (A + B) (0) \leq G_{\alpha,\beta} (A + B) (x)$

$\Omega_A (0). \Omega_B (0) \geq \Omega_A (x). \Omega_B (x)$

$G_{\alpha,\beta} \{\Omega_A (0). \Omega_B (0)\} \geq G_{\alpha,\beta} \{\Omega_A (x). \Omega_B (x)\}$

$\{\beta \Omega_A (0). \beta \Omega_B (0)\} \geq \{\beta \Omega_A(x). \beta \Omega_B(x)\}$

$\{\beta^2 \Omega_A (0). \Omega_B (0)\} \geq \{\beta^2 \Omega_A (x). \Omega_B (x)\}$

$G_{\alpha,\beta} (A + B) (0) \geq G_{\alpha,\beta} (A + B) (x)$

Case 2:

$\{ \Psi_A (x - (x - y)) + \Psi_B (x - (x - y)) - \Psi_A (x - (x - y)). \Psi_B (x - (x - y))\} \leq \max \{[\Psi_A ((y -(y - (x - (x - y))) - z), \Psi_A(z)]\} + \max [\Psi_B ((y - (y - (x - (x - y))) - z), \Psi_B (z)]- \max \{\Psi_A ((y - (y - (x - (x - y))) - z), \Psi_A (z)\}. \max \{\Psi_B ((y - (y - (x - (x - y))) - z), \Psi_B (z)\}\}$

$G_{\alpha,\beta} \{\Psi_A (x - (x - y)) + \Psi_B (x - (x - y)) - \Psi_A (x - (x - y)). \Psi_B (x - (x - y))\} \leq G_{\alpha,\beta} \max\{\{\mu_A ((y - (y - (x - (x - y))) - z) + \mu_B ((y - (y - (x - (x - y))), \mu_A(z) + \mu_B (z)\}- \max\{ \mu_A ((y - (y - (x - (x - y))) - z).\mu_B ((y - (y - (x - (x - y))), \mu_A (z).\mu_B (z)\}\}$

$\{\alpha\Psi_A(x-(x-y)) + \alpha\Psi_B(x-(x-y)) - \alpha\Psi_A(x-(x-y)). \alpha\Psi_B(x-(x-y))\} \leq \max \{\{\alpha \Psi_A ((y-(y-(x-(x-y)))-z) + \alpha \Psi_B ((y-(y-(x-(x-y)))-z), \alpha \Psi_A (z) + \alpha \Psi_B(z)\}- \max \{\alpha \Psi_A ((y-(y-(x-(x-y)))-z). \alpha \Psi_B((y-(y-(x-(x-y)))-z)-, \alpha \Psi_A (z). \alpha \Psi_B (z)\}\}$

$\{\alpha\Psi_A(x-(x-y)) + \alpha\Psi_B(x-(x-y)) – \alpha^2\Psi_A(x-(x-y)). \Psi_B(x-(x-y))\} \leq \max \{\{ \alpha \Psi_A ((y-(y-(x-(x-y)))-z) + \alpha\Psi_B ((y-(y-(x-(x-y)))-z) – \alpha^2\Psi_A ((y-(y-(x-(x-y)))-z). \Psi_B ((y-(y-(x-(x-y)))-z)\}, \{\alpha \Psi_A (z) + \alpha\Psi_B (z)- \alpha^2\Psi_A (z). \Psi_B (z)\}\}$

$G_{\alpha,\beta}(A+B) (x-(x-y)) \leq \max \{G_{\alpha,\beta} (A+B) ((y-(y-(x-(x-y)))-z), G_{\alpha,\beta} (A+B) (z)\}$

$\Omega_A (x - (x - y)). \Omega_B (x - (x - y)) \geq \min \{\Omega_A ((y - (y - (x - (x - y)))) - z), \Omega_A (z)\}. \min \{\Omega_B ((y - (y - (x - (x - y)))) - z), \Omega_B (z)\}$

Ω_A (x - (x - y)). Ω_B (x - (x - y)) ≥ min {Ω_A ((y - (y - (x - (x - y)))) - z). Ω_B ((y - (y - (x - (x - y))))-z}, min {Ω_A (z). Ω_B(z)}
$G_{\alpha,\beta}${ Ω_A(x - (x - y)). Ω_B (x - (x - y))}≥ $G_{\alpha,\beta}${ min { Ω_A ((y - (y - (x - (x - y)))) - z). Ω_B ((y - (y - (x - (x - y))))-z}, min{ Ω_A (z). Ω_B(z)}}
{β Ω_A(x - (x - y)). β Ω_B (x - (x - y))} ≥min {β Ω_A ((y - (y - (x - (x - y)))) - z). β Ω_B ((y - (y - (x - (x - y))))-z}, min {α Ω_A(z)).α Ω_B (z)}
{ β^2 Ω_A (x - (x - y)). Ω_B (x - (x - y))} ≥min {{β^2 Ω_A((y - (y - (x - (x - y)))) - z). Ω_B ((y - (y - (x - (x - y))))-z},{ β^2 Ω_A(z)).α Ω_B (z}}
$G_{\alpha,\beta}$(A+B)(x- (x - y))≥min{ $G_{\alpha,\beta}$(A+B)((y - (y - (x - (x - y)))), $G_{\alpha,\beta}$(A+B) (A+B)(z)}
Therefore, the two numbers of A+B is an antagonistic- intuitionistic fuzzy sub-commutative ideal in subtraction G-algebra of X.

Theorem 3.2. If $G_{\alpha,\beta}$(A) and $G_{\alpha,\beta}$(B) are antagonistic -intuitionistic fuzzy sub-commutative ideal of subtraction G-algebra then $G_{\alpha,\beta}$(A•B) is also antagonistic -intuitionistic fuzzy sub-commutative ideal of subtraction G-algebra.
Proof: Given $G_{\alpha,\beta}$(A) and $G_{\alpha,\beta}$(B) are antagonistic -intuitionistic fuzzy sub-commutative ideal of X.
(1) Ψ_A (0)≤ Ψ_A (x), Ω_A(0)≥Ω_A(x) for all x ∈ X, and
(2) Ψ_A(x - (x - y)) ≤ max { Ψ_A ((y - (y - (x - (x - y)))) - z), Ψ_A(z)} for all x, y, z ∈ X.
(3) Ω_A (x - (x - y)) ≥min { Ω_A((y - (y - (x - (x - y)))) - z), Ω_A (z))} for all x, y, z ∈ X.
(4) Ψ_B (0)≤ Ψ_B (x), Ω_B(0)≥ Ω_B(x) for all x ∈ X, and
(5) Ψ_B (x - (x - y)) ≤ max { Ψ_B ((y - (y - (x - (x - y)))) - z), Ψ_B (z)} for all x, y, z ∈ X.
(6) Ω_B (x - (x - y))≥ min { Ω_B ((y - (y - (x - (x - y)))) - z), Ω_B (z))} for all x, y, z ∈ X.
Case1:
{Ψ_A(0). Ψ_B(0)} ≤ {Ψ_A(x). Ψ_B (x)}
$G_{\alpha,\beta}${Ψ_A(0). Ψ_B(0)}≤ $G_{\alpha,\beta}${Ψ_A (x). Ψ_B (x)}
{αΨ_A(0). αΨ_B(0)}≤ { αΨ_A (x). αΨ_B (x)}
{$\alpha^2$$\Psi_A$(0). Ψ_B(0)}≤ { $\alpha^2$$\Psi_A$ (x). Ψ_B (x)}
$G_{\alpha,\beta}$(A•B)(0)≤ $G_{\alpha,\beta}$(A•B)(x)
{Ω_A(0)+ Ω_B(0) - Ω_A(0). Ω_B(0)} ≥ {Ω_A(x)+ Ω_B (x)- Ω_A (x). Ω_B (x)}
$G_{\alpha,\beta}${ Ω_A(0)+ Ω_B(0) - Ω_A(0). Ω_B(0)} ≥ $G_{\alpha,\beta}${ Ω_A(x)+ Ω_B (x)- Ω_A (x). Ω_B (x)}
{β Ω_A(0)+ β Ω_B(0) - β Ω_A(0). β Ω_B(0)} ≥{ β Ω_A(x)+ β Ω_B (x)- β Ω_A (x). β Ω_B (x)}
{β Ω_A(0)+ β Ω_B(0) – $\beta^2$$\Omega_A$(0). Ω_B(0)} ≥ { β Ω_A(x)+ β Ω_B (x)- $\beta^2$$\Omega_A$ (x). Ω_B (x)}
$G_{\alpha,\beta}$(A•B)(0) ≥ $G_{\alpha,\beta}$(A•B)(x)
Case2:
{Ψ_A(x- (x- y)). Ψ_B (x- (x- y))} ≤ max {Ψ_A ((y-(y-(x-(x-y)))-z), Ψ_A (z)}. max {Ψ_B ((y-(y-(x-(x-y)))-z), Ψ_B (z)}}
$G_{\alpha,\beta}${ Ψ_A(x-(x-y)). Ψ_B(x-(x-y))} ≤ $G_{\alpha,\beta}$ max{ Ψ_A ((y-(y-(x-(x-y)))-z). Ψ_B((y-(y-(x-(x-y))), Ψ_A(z). Ψ_B(z)}}
{αΨ_A(x-(x-y)). αΨ_B(x-(x-y))} ≤ max {α Ψ_A ((y-(y-(x-(x-y)))-z). α Ψ_B((y-(y-(x-(x-y)))-z)-, α Ψ_A (z). α Ψ_B (z)}}
{$\alpha^2$$\Psi_A$(x-(x-y)). Ψ_B(x-(x-y))} ≤ max {{$\alpha^2$$\Psi_A$ ((y-(y-(x-(x-y)))-z). Ψ_B ((y-(y-(x-(x-y)))-z)}, {α Ψ_A (z) + αΨ_B (z)- $\alpha^2$$\Psi_A$ (z). Ψ_B (z)}}

$G_{\alpha,\beta}(A\bullet B)(x-(x-y)) \leq \max\{ G_{\alpha,\beta}(A\bullet B)((y-(y-(x-(x-y)))-z), G_{\alpha,\beta}(A\bullet B)(z)\}$

$\{ \Omega_A(x-(x-y)) + \Omega_B(x-(x-y)) - \Omega_A(x-(x-y)).\ \Omega_B(x-(x-y))\} \geq \min\{[\Omega_A((y-(y-(x-(x-y)))-z), \Omega_A(z)]\} + \max[\Omega_B((y-(y-(x-(x-y)))-z), \Omega_B(z)] - \max\{\Omega_A((y-(y-(x-(x-y)))-z), \Omega_A(z)\}.\ \max\{ \Omega_B((y-(y-(x-(x-y)))-z), \Omega_B(z)\}\}$

$\{ \Omega_A(x-(x-y)) + \Omega_B(x-(x-y)) - \Omega_A(x-(x-y)).\ \Omega_B(x-(x-y))\} \geq \{\min\{ \Omega_A((y-(y-(x-(x-y)))-z) + \Omega_B((y-(y-(x-(x-y)))-z) - \Omega_A((y-(y-(x-(x-y)))-z).\ \Omega_B((y-(y-(x-(x-y)))-z)\}, \min\{\Omega_A(z) + \Omega_B(z) - \Omega_A(z).\ \Omega_B(z)\}\}$

$\Omega_A(x-(x-y)) + \Omega_B(x-(x-y)) - \Omega_A(x-(x-y)).\ \Omega_B(x-(x-y))\} \geq \{\min\{ \Omega_A((y-(y-(x-(x-y)))-z) + \Omega_B((y-(y-(x-(x-y)))-z) - \Omega_A((y-(y-(x-(x-y)))-z).\ \Omega_B((y-(y-(x-(x-y)))-z)\}, \{\Omega_A(z) + \Omega_B(z) - \Omega_A(z).\ \Omega_B(z)\}\}$

$G_{\alpha,\beta}\{\Omega_A(x-(x-y)) + \Omega_B(x-(x-y)) - \Omega_A(x-(x-y)).\ \Omega_B(x-(x-y))\} \geq \min G_{\alpha,\beta}\{\{\Omega_A((y-(y-(x-(x-y)))-z) + \Omega_B((y-(y-(x-(x-y))), \Omega_A(z) + \Omega_B(z)\} - \max\{\Omega_A((y-(y-(x-(x-y)))-z).\ \Omega_B((y-(y-(x-(x-y))), \Omega_A(z).\ \Omega_B(z)\}\}$

$\{\alpha\Omega_A(x-(x-y)) + \alpha\Omega_B(x-(x-y)) - \alpha\Omega_A(x-(x-y)).\ \alpha\Omega_B(x-(x-y))\} \geq \min\{\{\alpha\Omega_A((y-(y-(x-(x-y)))-z) + \alpha\Omega_B((y-(y-(x-(x-y)))-z), \alpha\Omega_A(z) + \alpha\Omega_B(z)\} - \max\{\alpha\Omega_A((y-(y-(x-(x-y)))-z).\ \alpha\Omega_B((y-(y-(x-(x-y)))-z), \alpha\Omega_A(z).\ \alpha\Omega_B(z)\}\}$

$\{\alpha\,\Omega_A(x-(x-y)) + \alpha\,\Omega_B(x-(x-y)) - \alpha^2\,\Omega_A(x-(x-y)).\ \Omega_B(x-(x-y))\} \geq \min\{\{\alpha\,\Omega_A((y-(y-(x-(x-y)))-z) + \alpha\,\Omega_B((y-(y-(x-(x-y)))-z) - \alpha^2\Omega_A((y-(y-(x-(x-y)))-z).\ \Omega_B((y-(y-(x-(x-y)))-z)\}, \{\alpha\Omega_A(z) + \alpha\Omega_B(z) - \alpha^2\Omega_A(z).\ \Omega_B(z)\}\}$

$G_{\alpha,\beta}(A \bullet B)(x-(x-y)) \geq \min\{G_{\alpha,\beta}(A \bullet B)((y-(y-(x-(x-y)))-z), G_{\alpha,\beta}(A \bullet B)(z)\}$

Therefore, the two numbers of A. B is an antagonistic- intuitionistic fuzzy sub-commutative ideal in subtraction G-algebra of X.

Example 3.1. Let X = {0, a, b, c} be a subtraction G-algebra with the following Cayley table.

-	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	a	b	0

We define an anti- fuzzy set A then A = < X, Ψ_A, Ω_A> by routine calculation A is an antagonistic -intuitionistic fuzzy sub-implicative ideal of X.

X	0	a	b	c
Ψ_A	.40	.42	.62	.75
Ω_A	.50	.45	.29	.21

We define an anti- fuzzy set B then B = < X, Ψ_B, Ω_B > by routine calculation B is an antagonistic -intuitionistic fuzzy sub-implicative ideal of X.

X	0	a	b	C
Ψ_B	.51	.53	.61	.72
Ω_B	.47	42	.31	.26

If A and B are antagonistic -intuitionistic fuzzy sub-implicative ideal of X. obviously the union A. B, Since x = b, y = c, z = a, α =.41, β=.46. $\{\alpha^2\Psi_A (0). \Psi_B (0)\} \leq \{\alpha^2\Psi_A(x). \Psi_B(x)\}$. 0343 ≤ .0636
$\{\beta\Omega_A (0) +\beta\Omega_B (0)-\beta^2\Omega_A (0). \Omega_B (0)\} \geq \{\beta\Omega_A(x)+\beta \Omega_B(x)- \beta^2 \Omega_A(x). \Omega_B(x)\}$. 4605 ≥ .2569
$\{\alpha^2 \Psi_A (x - (x - y)). \Psi_B (x - (x - y))\} \leq \max \{\{\alpha^2\Psi_A ((y - (y - (x - (x - y)))) - z). \Psi_B ((y - (y - (x - (x - y)))) - z)\}, \{\alpha^2\Psi_A(z). \Psi_B(z)\}\}$. 0920 ≵ .0636
$\{\beta\Omega_A (b - (b - c)) + \beta\Omega_B (b - (b - c))- \beta^2 \Omega_A (b - (b - c)). \Omega_B (b - (b - c)) \geq \min \{\{\beta\Omega_A ((c - (c - (b - (b - c)))) - a) + \beta \Omega_B ((c - (c - (b - (b - c)))) - a)- \beta^2 \Omega_A ((c - (c - (b - (b - c)))) - a). \Omega_B ((c - (c - (b - (b - c)))) - a)\}, \{\beta\Omega_A(a)) + \beta\Omega_B(a))- \beta^2 \Omega_A(a)). \Omega_B(a))\}\}$. 2047 ≵. 3603
Therefore, the given two products of antagonistic -intuitionistic fuzzy sub-implicative ideal of X is not an antagonistic -intuitionistic fuzzy sub-implicative ideal of X.

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