Ratio Mathematica Volume 47, 2023

Edge coloring in complement of bipolar fuzzy graphs

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Abstract

Graph theory is rapidly moving into the mainstream of mathematics because of its applications in diverse fields which include chemistry, bio-chemistry, electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). Graph coloring is one of the most important concepts in graph theory and is used in many real time applications like Job scheduling, Aircraft scheduling, computer network security, Map coloring, Automatic channel allocation for small wireless local area networks. Two types of coloring namely vertex coloring and edge coloring are usually associated with any graph. In this paper, we analyze edge coloring of the complement bipolar fuzzy graphs using concept of as bipolar fuzzy numbers through the $\alpha -$ cuts of bipolar fuzzy graphs. For different values of $\alpha -$ cuts which depend on edge and vertex membership value of the graph, we will get different graph and different chromatic number.

Keywords: complement bipolar fuzzy graph; edge coloring; chromatic number; $\alpha -$ cut of BFG.

2010 AMS subject classification: 05C72‡

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‡ Received on September 15, 2022. Accepted on December 15, 2022. Published on March 10, 2023. DOI:10.23755/rm.v39i0.969. ISSN: 1592-7415. eISSN: 2282-8214.

1. Introduction

In 1994, Zhang [13] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with bipolar fuzzy information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several direction. Akram[1] introduced the concept of bipolar fuzzy graphs and defined different operations on it. Sunil Mathew, M.S.Sunitha and A.Vijaya kumar[8] introduced the concept of complement bipolar fuzzy graphs in 2014 and discussed about some connectivity concepts in bipolar fuzzy graphs. As an advancement coloring of fuzzy graph was outlined by authors Eslahchi and Onagh[4] in 2004, and later developed by them a fuzzy vertex coloring in 2006. This fuzzy vertex coloring was extended to fuzzy total coloring in terms of family of fuzzy set by M.Ananthanarayanan and S.Lavanya [2]. The concept of chromatic number of fuzzy graphs was introduced by Munoz.S[6]. Anjali and Sunitha [3] developed algorithms to the chromatic number of fuzzy graphs. Types of Paths and Strong Cycle Connectivity in Bipolar Fuzzy Graphs discussed by S. Y.Mohamed and Subashini[11,12] A.Tahmasbpour and R.A.Borzooei[9] introduced the concept of chromatic number of bipolar fuzzy graphs. In this paper, we determine edge chromatic number of Complement bipolar fuzzy graphs using α −cut value.

2. Preliminaries

Definition 2.1

By a bipolar fuzzy graph, we mean a pair $G = (A, B)$ where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on E such that $\mu_B^P(xy) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all$(x, y) \in E$.

Definition 2.2

We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E respectively. Note that B is symmetric bipolar fuzzy relation on A. we use the notation xy for an element of E. Thus, G = (A,B) is a bipolar fuzzy graph of $G^* = (V, E)$ if $\mu_B^P(xy) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all $xy \in E$. $H = (\alpha, \beta)$ (where $\alpha = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy subset of a set A and $\beta = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on B) is called a partial bipolar fuzzy subgraph of G if $\alpha \le A$ and $\beta \le B$. We call $H = (\alpha, \beta)$ a spanning bipolar fuzzy subgraph of $G = (A, B)$ if $\alpha = A$.

Definition 2.3
An arc $\left(\left(\mu_A^p(u), \mu_A^N(u)\right), \left(\mu_A^p(v), \mu_A^N(v)\right)\right)$ of G is called m-strong if $\mu_B^P\,(u,v) = \min\,(\mu_A^P(u), \mu_A^P(v))$ and $\mu_B^N\,(u,v) = \max\,(\mu_A^N(u), \mu_A^N(v))$. Suppose G : (A,B) be a bipolar fuzzy graph. The complement of G is denoted as $\bar{G} : (\bar{A}, \bar{B})$ where $\bar{A} = A$ and $\bar{\mu}_B^P\,(xy) = \min(\mu_A^P(x), \mu_A^P(y)) - \mu_B^P(xy)$ and $\bar{\mu}_B^N\,(xy) = \max(\mu_A^N(x), \mu_A^N(y)) - \mu_B^N(xy)$.

Definition 2.4
$\propto -$ cut set of Bipolar fuzzy set A is denoted as $A_\propto$ is made up of members whose positive membership is not less than $\propto$ and negative membership is not greater than $\propto$. $A_\propto{}^P = \{x \in X\,, \mu_A{}^P(x) \geq \propto\}$ and $A_\propto{}^N = \{x \in X\,, \mu_A{}^N(x) \leq \propto\}$ where $\propto -$ cut set of fuzzy set is crisp set. The $\propto -$ cut of BFG defined as $G_\propto = (\,V_\propto, E_\propto)$ where $V_\propto = \{v \in V/\sigma \geq \propto\}$ and $E_\propto = \{e \in E/\mu \geq \propto\}$.

3. Complement of bipolar fuzzy graph

Complement of FG has been defined by Moderson[5]. Complement of a BFG $G : (\sigma, \mu)$ as a BFG $\bar{G} : (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\mu_{\bar{B}}{}^P(xy) = 0\ if\ \mu(xy) > 0$, $\mu_{\bar{B}}{}^N(xy) = 0\ if\ \mu(xy) < 0$ and $\mu_{\bar{B}}{}^P(xy) = min\,[\mu_A{}^P(x), \mu_A{}^P(y)]$, $\mu_{\bar{B}}{}^N(xy) = max\,[\mu_A{}^N(x), \mu_A{}^N(y)]$ otherwise. From the definition $\bar{G}$ is a BFG even if G is not and $\bar{\bar{G}} = G$ if and only if G is m-strong BFG. Also, automorphism graph of G and $\bar{G}$ are not identical. But there is some drawbacks in the definition of complement of a BFG mentioned above. In Fig 3.3, $\bar{\bar{G}} \neq G$ and note that they are identical provided G is m-strong BFG.

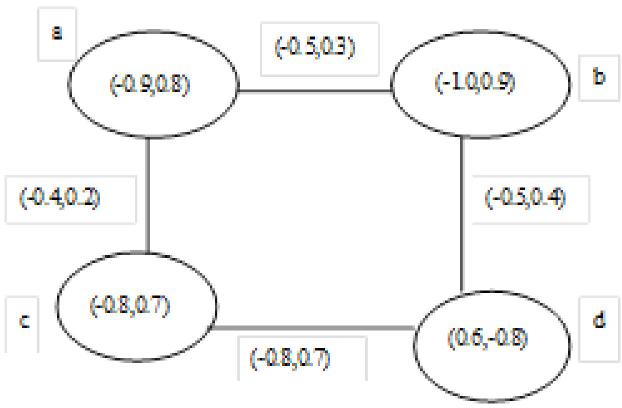

Figure 3.1: Bipolar fuzzy graph G

Now the Complement of a BFG $G : (\sigma, \mu)$ is the BFG $\bar{G} : (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\mu_{\bar{B}}{}^P(xy) = min\,[\mu_A{}^P(x), \mu_A{}^P(y)] - \mu_B{}^P(xy)$ and $\mu_{\bar{B}}{}^N(xy) = max\,[\mu_A{}^N(x), \mu_A{}^N(y)] - \mu_B{}^N(xy)$

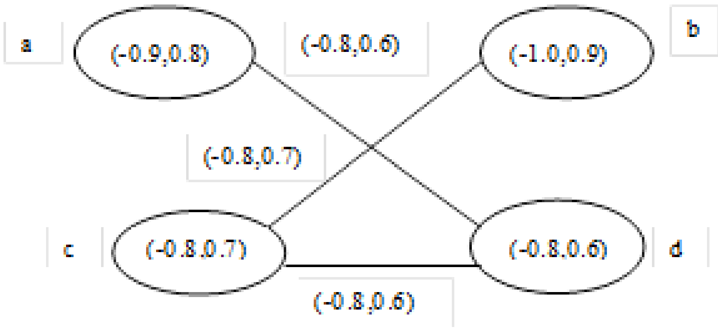

Figure 3.2: Complement of bipolar fuzzy graph $\bar{G}$

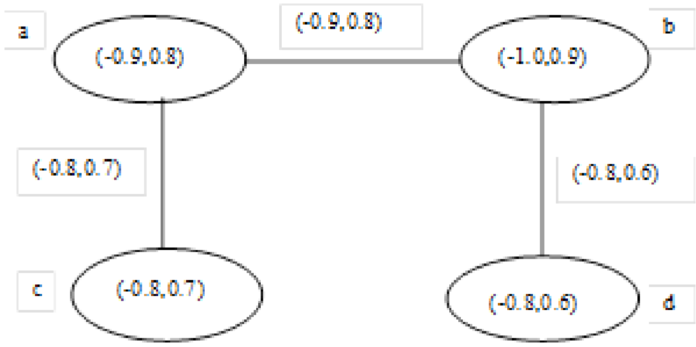

Figure 3.3: Complement of complement bipolar fuzzy graph $\bar{\bar{G}}$

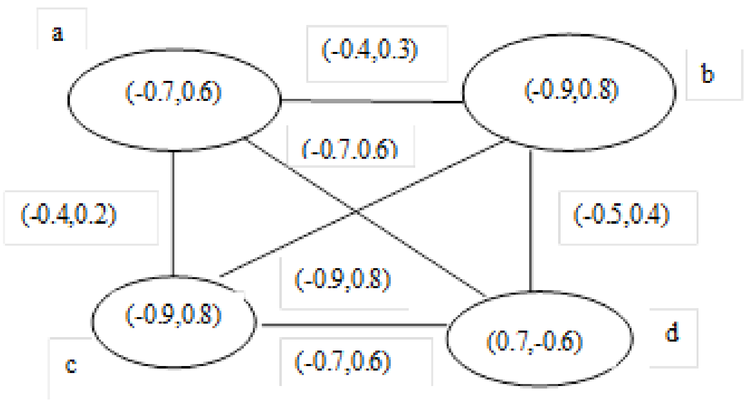

Figure 3.4: Bipolar fuzzy graph G

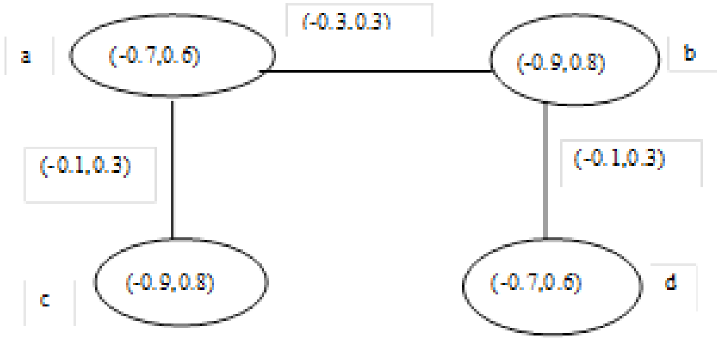

Figure 3.5: Complement of bipolar fuzzy graph $\bar{G}$

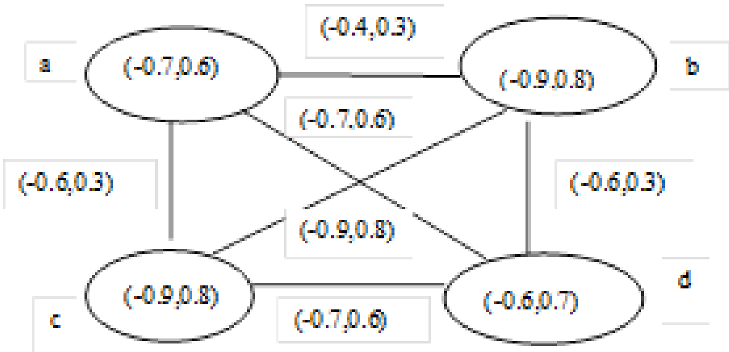

Figure 3.6: Complement of complement bipolar fuzzy graphs

Now $\bar{\sigma} = \sigma$ and $\mu_{\bar{B}}{}^{P}(xy) = min\ [\mu_A{}^P(x), \mu_A{}^P(y)] - \mu_B{}^P(xy)$ and
$$\mu_{\bar{B}}{}^{N}(xy) = max\ [\mu_A{}^N(x), \mu_A{}^N(y)] - \mu_B{}^N(xy).$$
Hence $\bar{\bar{G}} = G$. This shows that complement of complement BFG is a BFG.

4. Edge coloring in complement of BFG

We find all the different membership value of vertices and edges in the complement of a BFG. This membership value will work as a cut of this complement BFG. Depend upon the values of $\propto -$ cut we find different types of Bipolar fuzzy sets for the same complement BFG. Then we color all the edges of the complement BFG so that no incident edges will not get the same color and find the minimum number of color will need to color the complement BFG is known as chromatic number.

For solving this problem we have the calculation into three cases. In first case we take a BFG (G) which have 5 vertices and 5 edges. All the vertices and edges have membership value. In second case we find the complement of this graph (G_1). In third case we define the edge coloring function to color the complement BFG.

Case 1:

Consider a BFG which have five vertices a, b, c, d, e and corresponding membership values $\{(-0.8,0.9),(-0.8,0.75),(-0.85,0.95),(-0.85,0.95),(-0.8,0.9)\}$. Graph consist of five edges $e_1\, e_2, e_3, e_4, e_5$ with their corresponding membership value $\{(-0.8,0.75),(-0.8,0.9),(-0.6,0.85),(-0.85,0.95),(-0.4,0.6)\}$ corresponding BFG is shown in Fig 4.1.

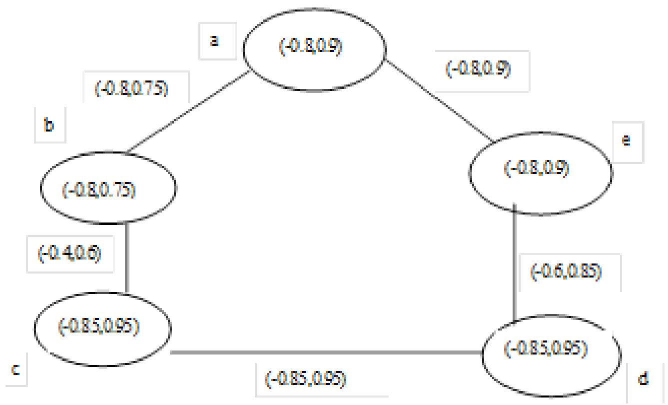

Figure 4.1: Bipolar fuzzy graph G

Case 2.

We find the complement of a BFG.

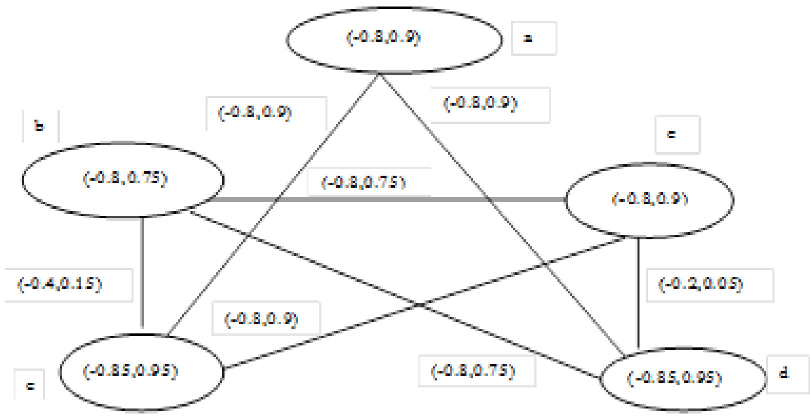

Figure 4.2: Complement of bipolar fuzzy graph $\bar{G}$

Case 3.

Given a BFG $G = (V, E)$ its edge chromatic number is bipolar fuzzy number $\chi(G) = \{(X_\alpha, \alpha)\}$ where X_α is the edge chromatic number of G_α where α values are the different membership value of vertex and edge of graph G. In this BFG there are five α −cuts. There are $\{(-0.2,0.05),(-0.4,0.15),(-0.8,0.75),(-0.8,0.9),(-0.85,0.95)\}$. For every value of α, we find graph G_α and find its fuzzy edge chromatic number.

For $= (-0.2,0.05)$, BFG $G = (\sigma,\mu)$ where $\sigma = \{(-0.8,0.9),(-0.8,0.75),(-0.85,0.95),(-0.85,0.95),(-0.8,0.9)\}$.

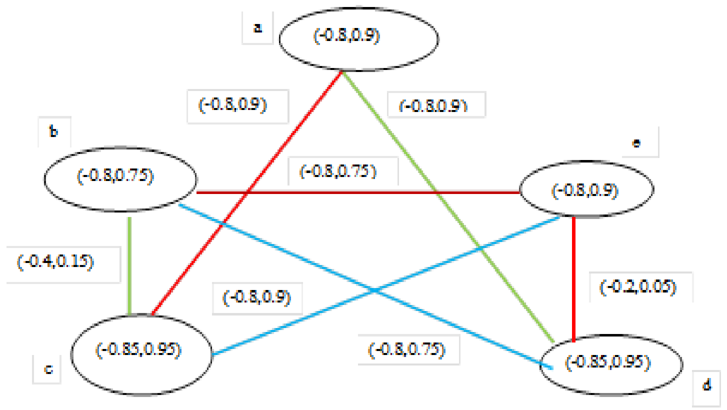

Figure 4.3: $\chi(-0.2,0.05) = 4$

For $\alpha -$ cut (-0.2,0.05) we find the graph $G_{(-0.2,0.05)}$ (Figure 4.3). Then we proper color all the edges of this graph and the chromatic number of this graph is 4.

For $\alpha = (-0.4,0.15)$ BFG $G = (\sigma,\mu)$ where $\sigma = \{(-0.8,0.9),(-0.8,0.75),(-0.85,0.95),(-0.85,0.95),(-0.8,0.9)\}$

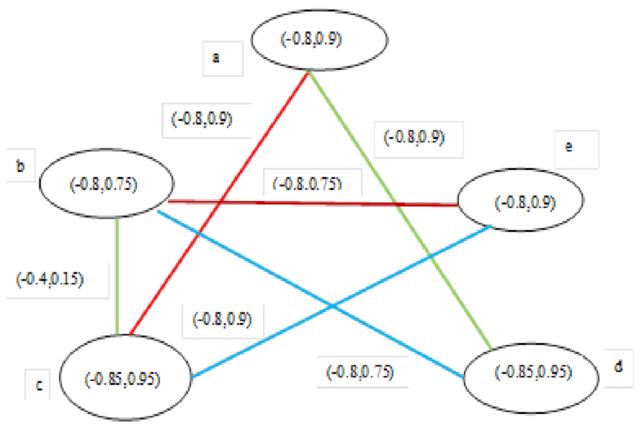

Figure 4.4: $\chi(-0.4,0.15) = 4$

For $\alpha -$ cut value (-0.4,0.15) we find the graph $G_{(-0.4,0.15)}$ (Figure 4.4). Then we proper color all the edges of this graph and the chromatic number of this graph is

4(Four). For $\alpha = (-0.8,0.75)$ BFG $G = (\sigma, \mu)$ where $\sigma = \{(-0.8,0.9), (-0.8,0.75), (-0.85,0.95), (-0.85,0.95), (-0.8,0.9)\}$.

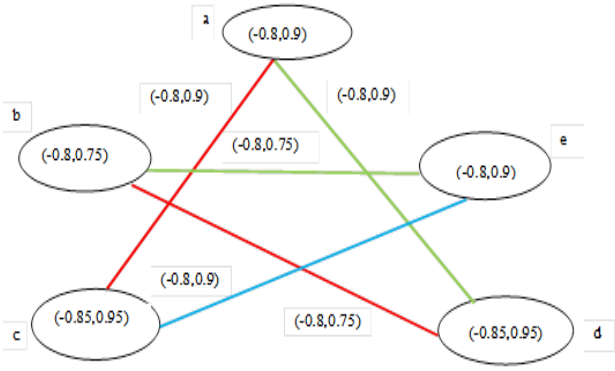

Figure 4.5: $\chi(-0.8,0.75) = 3$

Now for $\alpha -$ cut value (-0.8,0.75), we find the graph $G_{(-0.8,0.75)}$ (Figure 4.5). Then we proper color all the edges of this graph and the chromatic number of this graph is 3.

For $\alpha = (-0.8,0.9)$ BFG $G = (\sigma, \mu)$ where $\sigma = \{(-0.8,0.9), (-0.85,0.95), (-0.85,0.95), (-0.8,0.9)\}$.

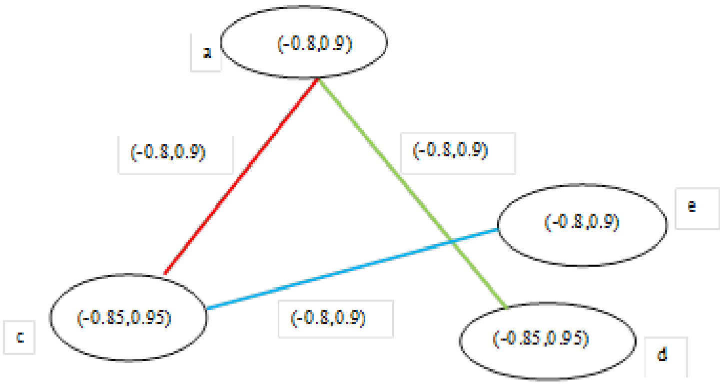

Figure 4.6: $\chi(-0.8,0.9) = 2$

For $\alpha -$ cut value (-0.8,0.9) we find the graph $G_{(-0.8,0.9)}$ (Figure 4.6). Then we proper color all the edges of this graph and the chromatic number of this graph is 2.

For $= (-0.85,0.95)$, BFG $G = (\sigma, \mu)$ where $\sigma = \{(-0.85,0.95)\}$ and

Figure 4.7: $\chi(-0.8,0.9) = 0$

For $\alpha -$ cut value (-0.85,0.95), we find the graph $G_{(-0.85,0.95)}$ (Figure 4.7). Then we proper color all the edges of this graph and the chromatic number of this graph is 0.

In the above example five crisp graph $G_\alpha = (V_\alpha, E_\alpha)$ are obtained by considering different values of α. Now for the edge chromatic number χ_α for any α, it can be shown that the chromatic number of BFG is $\chi(G) =$
$\{(4,-0.2,0.05),(4,-0.4,0.15),(3,-0.8,0.75),(2,-0.8,0.9),(0,-0.85,0.95)\}$.

4 Conclusions

In this paper, we have found the complement bipolar fuzzy graph and color all the edges of that complement BFG through the $\alpha -$ cuts. Also, We observed that edge chromatic number depends on $\alpha -$ cut value.

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