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Elementary School Teachers’ Mathematical Connections in Solving Trigonometry Problem

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Abstract

This study aims to reveal mathematical connections of elementary school teachers in solving trigonometric problem. The subjects of this study were 22 elementary school teachers as the prospective participants of Professional Teacher Education and Training (PTET). They came from several districts of South Sulawesi Province. The teachers were given trigonometry problem. Trigonometry problems could encourage teachers to connect geometrical and algebraic concept, graphical representation and algebraic representation, as well as daily life context. The result shows that most of the subject teachers of this study solved the problem according to procedures they know without considering everyday life context. On the other hand, there were some subjects who connected problem with everyday life context using graphical, verbal, or numerical representation. Thus, subjects who were able to connect problem information with appropriate concepts and procedures are categorized as substantive connections. While the subjects who were able to connect problem information with mathematical concepts but less precise in using the procedure are categorized as classification connections.

Key words: *Mathematical connection, trigonometry problem, elementary school teachers*

Introduction

Mathematics is a network of interconnected ideas, not a collection of separate nodes, although it is often taught separately (Businskas, 2008; NCTM, 2000). Mathematical connections are one of the five process standards recommended by NCTM (2000) in addition to problem solving, reasoning and verification, communication, and representation. Mathematics is a collection of interconnected mathematical ideas that mutually build or support each other (Erbilgin, 2017).

A person who is able to connect mathematical ideas, facts, procedures, and relationships indicates that the person has deep mathematical understanding and that his/her knowledge is long-lasting

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(Eli et. al., 2011; Mhlolo et. al., 2012; NCTM, 2000; Saminanto & Kartono, 2015). In addition thereto, a learning that involves connections will not only make learners understand mathematics deeper but also make use of mathematics in everyday life (NCTM, 2000; Saminanto & Kartono, 2015). When someone learns something, the person is also in the process of connecting the new information with his prior knowledges (Businskas, 2008; Eli et. al., 2011; NCTM, 2000; Saminanto & Kartono, 2015). However, when the teachers help their students to make connections, they have helped their students to learn on how to think mathematically (NCTM, 2000).

As a matter of fact, teachers have an important role to create a learning experience that allows students to recognize and interpret mathematical connections (Mhlolo et. al., 2012). Moreover, teachers who do not understand connections between important functional concepts in mathematics will be unable to engage students effectively to make connections, reasoning, and solve problem (Eli et. al., 2011; Saminanto & Kartono, 2015).

Connecting two or more mathematical ideas is a cognitive process involving the activity of creating or recognizing the interrelationships between the ideas. Mathematical connections can manifest the interrelationships between concepts in a single topic of mathematics, interrelationships between topics in mathematics, mathematical interrelation with other disciplines or daily life (Businskas, 2008; Kilinc, 2015; Mhlolo et. al., 2012; NCTM, 2000; Saminanto & Kartono, 2015; Tarman, 2010; 2017; Tarman & Chigisheva, 2017). Mousley (2004) categorizes mathematical connections into three, i.e., the connections that students construct between new information and their existing knowledge, the relationship between different mathematical ideas and representations, and the connections made between mathematics learned in schools and the mathematical aspects found in everyday life context.

The topic of trigonometry in mathematics blends the visual and symbolic aspects of geometry and algebra (Chin, 2013). Solving trigonometric problems requires the mastery of certain concepts of geometry and algebra, as well as the ability to connect them. In trigonometry, the use of graphical representations and algebraic representations are mutually supportive. Therefore, elementary school teachers need to have the ability to solve trigonometric problems, even though trigonometry is not taught in elementary school.

Based on the reason mentioned above, this study is aimed to reveal the mathematical connections of elementary school teachers in solving trigonometric problems.

Theoretical Framework

There have been some studies conducted previously which results in categorization of connections based on certain aspects. Mousley (2004) categorizes mathematical connections based on their scope, the connection between new information and prior knowledge, the connection between different mathematical ideas and representations, and the connection of mathematical ideas in everyday life context. Eli et. al. (2011) examined on the prospective teachers' views of the interrelationships between mathematical ideas, i.e. categorical, characteristic, curricular, procedural, and derivational. Arjudin et. al. (2014) highlighted problem-solving process, i.e., classification connections, substantive connections, and expansive connections.

According to Arjudin (2017), classification connection arises when a person is able to connect the problem information with mathematical concepts but does not connect with procedures related to mathematical concepts and the problem solving. The person knows the mathematical concepts needed to solve the problem, but is unable to connect between concepts or does not master the procedure to solve the problem. On the other hand, substantive connection occurs when a person is able to connect the problem information with mathematical concepts and connect the mathematical concepts with the procedures he/she mastered. The last, expansive connection occurs when a person is able to connect problem information with mathematical concepts and procedures and can build new procedures that are connected with problem solving. Expansive connection is done when someone solves a completely new problem for him/her. The person will connect the problem information to be solved with information from previously solved problems. From that connection will then form a new procedure to solve the problem.

Method

Participants

The study involved 22 elementary school teachers from several districts in South Sulawesi Province. The teachers are prospective participants of Professional Teacher Education and

Training (PTET), which is a government program to improve the quality of teachers so that they deserve a certificate of educators. Prior to the training, the teachers were asked to study independently of the modules available online. This module contained mathematics topics and exercises specially designed to refresh the teacher's knowledge. Teachers could consult online with instructors who had been determined. They were required to submit a report in the form of a summary of material that had been learned and answered the instructor's questions. The instructor would then assess and determine the teacher's skills that need to be improved.

Data Collection

The teachers were given the opportunity to study modules independently and work on practice questions from various mathematical topics. The study focused on the connections that teachers made in solving the following trigonometric problem:

> “An elementary student will determine the height of the flagpole without measuring directly. He stood at 12 m from the flagpole. Using a clinometer, it is known that the elevation angle between the student's eyes and the point of the flag pole is 60°. Determine the height of the flagpole!
>
> Calculate the distance of the eye to the ground!

Problem solving made by teacher become data in this research.

Data Analysis

The teachers' answers were distinguished by the appearance of connections in everyday life context. There were two kinds of answers, namely the answer that calculated the distance of the observer's eye to the ground (P) and the answer that did not calculate the distance (Q). Furthermore, the answers of group P were classified into categories of classification connection and substantive connection. The proposed problem does not allow any introduction of new procedures, so that expansive connection cannot be detected. Thus, expansive connection was not used in this study.

Findings

In the trigonometry problem proposed, the teachers were asked to determine the height of the flagpole by utilizing the comparison of trigonometry. Teachers who are able to make connections between problems with the context in everyday life would certainly calculate the distance of the eye of the observer to the ground. Although there is no information related to the distance in the question, teachers may use variables, verbal sentences, or make assumptions.

Based on the results of the answer analysis of 22 teachers, only 9 people calculated the distance of observer's eyes to the ground (group P). The other 13 teachers did not calculate the observer's eyes distance to the ground (group Q). Group Q solved problems according to the procedures they know, regardless of everyday life context. They did not realize that the answers they provide had not shown the actual flagpole. There was still a part of the flagpole neglected, which is the distance of the observer's eye to the ground.

Group P

There were 9 teachers who calculated the distance of observer's eye to the ground (group P). Four teachers described sketches with observer distance to the ground, two teachers drew sketches but incomplete, and three teachers did not draw sketches. The teacher used verbal sentences, variables, or assumptions to show the observer's eye distance to the ground.

P04 used the y symbol on the sketch and description to show the observer's eye distance to the ground. The final answer of P04 is "Thus, the entire height of the flagpole is the observer's height + AB = y + 12 $\sqrt{3}$". Figure 1 shows the sketch made by P04.

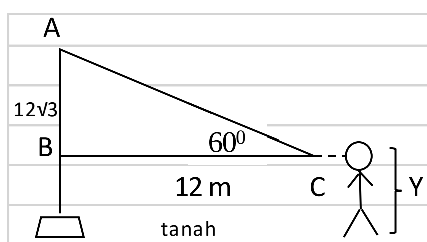

Figure 1. P04's sketch

P11 and P21 used the p symbol on the sketch and made assumptions. P11 and P21 each estimated the observer's eye distance to ground 1.52 meters and 1.5 meters. No further explanation had been made on the basis of these estimations, whether they connected it with the ideal height of high school students or not. P21 performed a calculation error in $12\sqrt{3} + 1{,}5 = 13{,}5\sqrt{3}$. This results in P21's incorrect answer. Figure 2 shows the sketch made by P21.

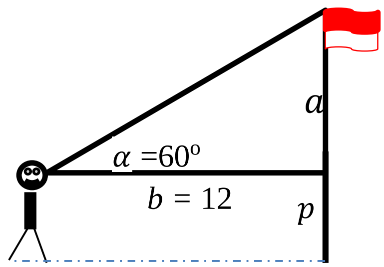

Figure 2. P21's sketch

Meanwhile, P22 did not use symbols either in sketches or descriptions of answers to indicate the distance of the observer's eye to the ground. P22 used verbal sentences. Here is an excerpt of the final answer of P22.

> Thus, the height of the flagpole from the eye distance is $12\sqrt{3}$.
>
> If calculated from the ground, then the height is $12\sqrt{3}$ plus the height of the student (up to the eye).

On the other hand, P09 and P15 made sketches but incomplete. They did not depict the line to show the observer's distance from the ground. Nevertheless they still calculated the distance. Figure 3 shows the sketch made by P15.

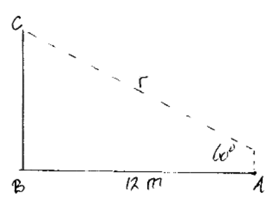

Figure 3. P15's sketch

P15 used verbal sentence in the answer, just like P22. P09 had a problem on the calculation, so the final answer is incorrect. Figure 4 shows the error P09 in performing the calculation.

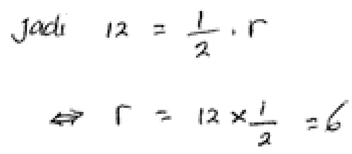

Figure 4. P09's calculation error

P05 did not make sketch, yet was able to rewrite the information in the problem and used variables. P05 made assumptions of the observer's height by connecting to knowledge or everyday experience. In the process of calculation, P05 mistakenly implemented the cosine concept so that the final answer is incorrect.

Besides, P02 and P16 did not sketch the problem, but directly used the trigonometric ratio to determine the height of the flagpole. They did not give explanation of the variables used. Thus, in order to obtain information on how they did it correctly, an interview is needed.

P02, P04, P11, P16, and P22 were able to connect information on problems with appropriate concepts and used appropriate procedures. Problem solving of the five subjects is categorized as substantive connection. Problem solving of the subjects P05, P09, P15, and P21 are categorized as classification connections. The four subjects were using incorrect procedures and which resulting on incorrect answers.

P04, P11, P15, and P22 were able to sketch based on information in the problem. It means they could connect verbal representations with graphical representations. P05, P11, and P21 used numerical representation that is 1.5 as "observer's eye distance to the ground". This representation is connected to the most students' height they know. P02, P04, P09, P15, P16, and P22 did not specify the distance of the observer's eye to the ground. P04 used the y variable to represent the distance, while the other subject used verbal representation.

Group Q

There were 13 teachers who did not calculate the distance of the observer's eye to the ground (group Q) in solving the trigonometric problem. Q08 made a complete sketch, but did not calculate the distance of the observer's eye to the ground in the answer. Q08 could well represent verbal problems into image form, but failed to connect the image with the final answer. More interviews were needed to find out if Q08 was able to relate problems with daily life experiences. If Q08 only calculated the x value in the image as the flagpole height, then the flagpole is in a floating position. Figure 5 shows the sketch made by Q08.

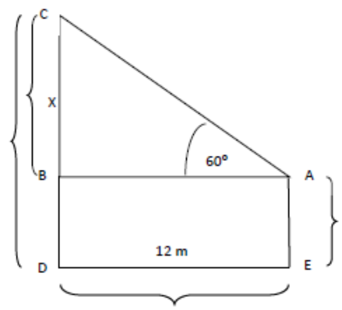

Figure 5. Q08's sketch

Q01, Q07, Q10, Q12, Q17, and Q18 made a sketch, but the sketch did not depict the observer's eye distance to the ground. The final answer obtained also did not involve that distance. Q03, Q06, Q13, Q14, Q19 and Q20 did not sketch the problem. They directly deciphered the answers involving the comparison of trigonometry and did not calculate the distance of the observer's eye to the ground.

Q19 paraphrased information about the distance of the observer's eye to the ground by writing as follows.

> "The height of the flagpole by calculating the distance of the observer's eye to the ground = ... m"

This paraphrase shows Q19 understood the problem, although at the end of the answer the information is not used.

Conclusion

The trigonometric problems proposed in this study provide opportunity for teachers to make assumptions based on their experience, particularly in connection to "observer's eye distance to ground". Group P calculated the distance, while group Q did not calculate the distance. There is still a need for in-depth interviews with subjects in group Q to obtain information why they did not calculate the distance.

Group P was able to connect information in the problem to their prior knowledge by using various representations, and was able to connect to everyday life context. In the P group, it is detected that there are the category of classification connection and substantive connection.

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