Research in Social Sciences and Technology

A COMPARISON OF THE MATHEMATICAL PROCESSES EMBEDDED IN THE CONTENT STANDARDS OF TURKEY AND SINGAPORE

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Abstract

This study compares Turkey's and Singapore's mathematics content standards in terms of the highligthed mathematical processes. A mathematical processes framework was employed to analyze the content standards drawing on the standards for mathematical practice defined by the Common Core State Standards for Mathematics. The standards for mathematical practice include make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, look for and express regularity in repeated reasoning. The data sources are 2013 mathematics curriculum standards of Turkey and 2013 mathematics syllabus of Singapore for grades 7 and 8. Data analysis revealed that the two countries reflected mathematical processes differently in their content standards. Some mathematical processes are not identified in Turkey's content standards while all mathematical processes are observed in Singapore's content standards. The distribution of the observed mathematical processes are also different in the two countries. Suggestions for future content standards revisions are shared in the paper.

Introduction

Curriculum standards are one of the factors that affect student learning and achievement through influencing what and how topics are to be taught in classrooms (Goertz, 2010; Pang, 2009). In the domain of mathematics, some curriculum standards are categorized into content standards and process standards (National Council of Teachers of Mathematics [NCTM], 2000). Content standards define what mathematical topics students are expected to learn, whereas process standards guide how students acquire mathematical knowledge. Process standards define what counts as a mathematical activity. For instance, according to NCTM

(2000), doing mathematics means formulating and solving complex problems, developing and testing mathematical conjectures, sharing and discussing mathematical ideas, recognizing and making connections among mathematical topics, and creating and using multiple representations. This type of mathematical practice is essential for developing students' conceptual understanding of mathematics (Hiebert, 2003), therefore, process standards might contribute to increased student learning in schools. Despite the importance of process standards in promoting student performance, they are often ignored in research studies that focus on analyzing or comparing curriculum standards (Tran, Reys, Teuscher, Dingman, & Kasmer, 2016).

There have been many research studies that compared curriculum standards of different countries (Porter, McMaken, Hwang, & Yang, 2011; Schmidt, Wang, & McKnight, 2005). Particular attention has been given to higher achieving countries whose students outperformed their peers in other countries in international studies such as Trends in International Mathematics and Science Study [TIMSS] and the Program for International Student Assessment [PISA]. Many of the research studies on mathematics curriculum comparison focused on analyzing and comparing what mathematics is included and in which sequence the topics are ordered within the content standards (Tran et al., 2016). Lacking are studies that investigate how mathematics topics are introduced in content standards. The current study aims to compare content standards of Turkey and Singapore in terms of process standards by examining how students are expected to learn mathematical topics as expressed in the content standards. The reason for choosing Turkey is the author's interest; she is from Turkey and aims to contribute to curriculum studies in her country. The reason for the choice of Singapore is that Singapore is among the top performing countries both in the most recent TIMSS (Provasnik, Malley, Stephens, Landeros, Perkins, & Tang, 2016) and PISA (The Organisation for Economic Co-operation and Development, 2016), and its curriculum documents are available online in

English. Comparing the two countries' curricula with a focus on mathematical processes will contribute to research efforts in the area of curriculum by providing a framework for analysis. There are frameworks for analyzing or comparing content standards based on their cognitive complexity (e.g., Depth of Knowledge, Webb (2007)) or content coverage (e.g., General Topic Trace Mapping (Schmidt et al., 2005), however, there is a need to develop a framework to analyze content standards with a focus on mathematical processes (Tran et al., 2016). The current study will also provide data to curriculum developers in Turkey, Singapore, and other countries as they consider integrating process standards into the content standards.

Theoretical Framework

In mathematics education literature, *how* students learn mathematical topics have been viewed as important as *what* mathematics they learn (Australian Association of Mathematics Teachers [AAMT], 2006; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000, 2014). Therefore many curriculum documents include mathematical processes as a guideline to implement content standards in classrooms. In this section, first I will review prominent mathematical processes from the related literature and then present the framework used in this study.

According to NCTM (2000), *mathematical process* standards include problem solving, reasoning and proof, connections, communication, and representations. *The problem solving* standard recommends that students construct new mathematical knowledge through solving challenging questions whose solution methods are not known in advance. Students should be encouraged to reflect on their problem solving processes and to develop a habit of mind that is characterized by curiosity and inquiry. *The reasoning and proof* standard suggests that students look for, develop, and test mathematical conjectures. They should also be able to defend and evaluate mathematical arguments, and justify mathematical results. This standard is about seeing mathematics as a discipline that is meaningful and logical. *The connections* standard advocates that mathematics is a collection of interconnected topics and mathematical concepts build on one another. Additionally, mathematics is related to other disciplines such as science and art. Such view of mathematics helps students understand mathematics conceptually and have a robust learning. *The communication* standard is about providing students with opportunities to share their mathematical thinking and reasoning with other students orally or in writing. Quality listening is also part of this standard. Mathematical communication promotes sharing of multiple perspectives and therefore is essential for rich learning. *The representations* standard involves expressing mathematical ideas in variety forms including words, graphs, tables, and equations. Students should be given opportunities to use multiple representations and translate one representation into another.

Kilpatrick et al. (2001) defined *mathematical proficiency* that is necessary for students to learn mathematics successfully. Mathematical proficiency has five interwoven and interdependent components: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. *Conceptual understanding* refers to knowing the reasons behind mathematical principles or formulas. A learner with conceptual understanding has a connected knowledge network. *Procedural fluency* means performing mathematical procedures accurately and effectively. *Strategic competence* is related to problem solving standard of NCTM (2000). It means being able to formulate and solve mathematical problems strategically. *Adaptive reasoning* indicates logical thinking about mathematical concepts. Similar to the reasoning and proof standard (NCTM, 2000), it provides students with capacity to analyze a mathematical situation critically, to reason both inductively and deductively, to reflect on their mathematical activity, and to justify mathematical arguments. *Productive disposition* toward mathematics is about valuing mathematics and seeing it useful in our lives.

More recently, the Common Core State Standards for Mathematics [CCSSM] ((National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) defined 8 standards for mathematical practice drawing on earlier work of NCTM (2000) and Kilpatrick et al. (2001). *Make sense of problems and persevere in solving them* refers to analyzing a problem situation carefully, develop and use a variety of solution strategies, and reflect on the problem solving progress. *Reason abstractly and quantitatively* is about contextualizing mathematical expressions and decontextualizing a given situation. *Construct viable arguments and critique the reasoning of others* indicates constructing and defending mathematical arguments and communicating them to others. *Model with mathematics* means mathematizing real world situations by creating a mathematical model and revising the model if necessary. *Use appropriate tools strategically* refers to selecting and using appropriate materials for solving a mathematics problem or investigating a mathematical idea and being aware of the limitations of available tools. *Attend to precision* is related to being clear and explicit about the definitions, symbols, graphs, and other forms of representations that are used. It is also related to computing fluently. *Look for and make use of structure* indicates finding patterns and using known structures to view a new situation with a different perspective. *Look for and express regularity in repeated reasoning* means concluding generalizations through analyzing individual cases and calculations.

The current study develops a mathematical processes framework to analyze content standards drawing on the standards for mathematical practice defined by CCSSM. The reason for choosing the mathematical processes defined by CCSSM is that it is comprehensive, based on previous frameworks, and detailed. I took each process standard defined by CCSSM and elaborated it such that it could be used to examine content standards. Table 1 contains the mathematical processes coding framework developed and used in this study. For each mathematical process, an example from each country's standards (when exists) is provided.

Table 1

Mathematical Processes Coding Framework

1. Make Sense of Problems and Persevere in Solving Them (PS)	
Problem Solving (PS)	The content standard expects students to Analyze the problem situation and plan a solution method. Solve problems in any strand of mathematics. Interpret the solution and compare different solution methods. Reflect on the problem solving progress.
Example:	7.3.2.5. Solves problems related to area. (Turkey) N2.c. …use algebra to solve problems,... (Singapore)
2. Reason Abstractly and Quantitatively (RAQ)	
Reason Abstractly (RA)	The content standard expects students to Represent a given situation symbolically. Make sense of formulas.
Example:	N7.c. Formulate inequalities from real-world contexts. (Singapore)
Reason Quantitatively (RQ)	The content standard expects students to Make sense of quantities in real world contexts. Estimate quantities (numbers, measurements). Compare quantities using units. Examine relationships between quantities. Make inferences from data.
Example:	8.1.3.3. Determine between which two whole numbers the value of a square root of a number that is not a perfect square lies. (Estimation activities to determine the closest whole number is suggested.) (Turkey)
3. Construct Viable Arguments and Critique the Reasoning of Others (CACRO)	
Construct Arguments (CA)	The content standard expects students to Explain why a mathematical statement is true or false. Elaborate the difference or similarity between mathematical concepts. Explain why a mathematical procedure works.
Example:	S1.b. Predict, observe and explain how the different measures of central tendency are affected by extreme data values (or outliers). (Singapore)
Critique the Reasoning of Others (CRO)	The content standard expects students to Judge whether other's mathematical arguments are valid or not. Discuss whether a mathematical argument is valid or not. Share and compare problem solution strategies. Discuss misconceptions. Work in groups to explore mathematical concepts. Discuss applications of mathematical topics in real life.
Example:	N3.d. Discuss misconception, e.g. "If A is 5% more than B, then B is 5% less than A." (Singapore)

4. Model with Mathematics (M)	
Modeling (M)	The content standard expects students to Construct a mathematical model to solve real life problems.
Example:	G8.a. Work on tasks that incorporate some or all elements of the mathematical modelling process (solving problems in real-world contexts using geometry). (Singapore)
5. Use Appropriate Tools Strategically (UT)	
Use Single Tool (UST)	The content standard expects students to Use a representation (tables, graphs, visual drawings, and symbols) to express mathematical ideas. Use concrete materials to make observations. Use technology to make mathematical investigations.
Example:	7.1.2.1. Identifies rational numbers and represents them on number line. (Turkey)
Use Multiple Tools (UMT)	The content standard expects students to Use multiple tools to investigate a mathematical concept (tools should be different structurally, for example, if a concrete material and its digital form is included in a standard, it is not coded under this category). Make connections among multiple representations (only translating one representation into another is not coded here. Connection between representations should be emphasized).
Example:	N6.d. Use a spreadsheet of graphing software to study how the graph of y=ax+b changes when either a or b varies. (Singapore)
6. Attend to Precision (AP)	
Attend to Accuracy (AA)	The content standard expects students to Use accurate labels, terminology, and notations. Describe situations with appropriate mathematical language. Attend to precision in measures and specify units.
Example:	8.2.2.1. …Identifies dependent and independent variables and examines how they change with respect to each other. (Turkey)
Attend to Fluency (AF)	The content standard expects students to Perform procedures fluently. Calculate efficiently and correctly.
Example:	N1.d. …develop proficiency in the 4 operations of integers. (Singapore)
7. Look for and Make Use of Structure (MUS)	
Make Use of Structure to Analyze (MUSA)	The content standard expects students to Analyze the structure of mathematical objects (e.g. even and odd numbers). Express a number, quantity, or mathematical expression in a different form (e.g. use properties of operations to rewrite algebraic expression). Classify mathematical objects based on their form.

Example:	Compare different forms of expressions including misconceptions. Analyze the properties of geometric shapes and solids.
Make Use of Structure to Make Connections (MUSC)	8.2.1.4. Factorize algebraic expressions. (Turkey)
	The content standard expects students to
	Make connections among mathematical topics.
Example:	Use real life examples to examine structure of mathematical objects.
	N3.c. Make connections between percentages and fractions/decimals, e.g. … (Singapore)
8. Look for and Express Regularity in Repeated Reasoning (ER)	
Express Regularity (ER)	The content standard expects students to
	Analyze and express patterns.
	Write patterns with symbols.
	Make generalizations.
	Discover rules and formulas.
Example:	8.3.4.3. Construct the surface area formula for right cylinders;… (Turkey)
	N5.f. Explore number patterns and write algebraic expressions to represent the patterns. (Singapore)

By using the framework presented in Table 1, the current study sought to answer the following research question:

- To what extent do opportunities to engage in mathematical processes in middle grades content standards of Turkey and Singapore vary?

Methodology

The set of curriculum standards is one of the main data sources in educational studies (Bogdan & Biklen, 1992). The middle school mathematics curriculum standards of Turkey and Singapore are the data sources of this study (Ministry of National Education [MoNE], 2013; Curriculum Planning and Development Division, 2012). Before explaining the data analysis process, some background information about each set of standards will be provided.

In Turkey, middle schools cover grades 5 through 8 and there is centralized curriculum governed by MoNE. The latest curriculum revision took place in 2013. The 2013 curriculum was examined in this study. In Turkey's mathematics curriculum standards, there are 5 content strands: numbers and operations, algebra, geometry and measurement, data analysis, and probability. For each grade level, the content standards are organized according to these strands. Some content standards include remarks and examples.

Singapore has 6 years of primary school education and 4-5 years of secondary education. Secondary school students enroll in Express, Normal (Academic), or Normal (Technical) courses of study with the following percentages of student enrollment: 60%, 25%, and 15% respectively (Mullis, Martin, Goh, & Cotter, 2016). The mathematics content standards of Normal (Academic) route are subset of the content standards of the Express route. Since more students enroll in Express course and most of its content standards are common with that of Normal (Academic) course, the mathematics content standards of the Express course are examined in the current study. Singapore's mathematics curriculum standards are organized around 3 content strands: number and algebra, geometry and measurement, and statistics and probability. At each grade level, content standards are presented in a table with two columns. One column shows content list, occasionally including remarks and examples. The other column shows corresponding learning experiences considered as content standards in the current study.

The first two years of secondary school (secondary one and secondary two) of Singapore correspond to the last two years of middle school (grades 7 and 8) of Turkey and these two grade levels have been the subject of this study. To avoid confusion, grade 7 and grade 8 will be used to indicate the grade levels for both countries. Hence, the data sources of this study include 2013 mathematics content standards of Turkey and Singapore for grades 7 and 8.

I conducted a content analysis of both set of standards using the mathematical processes coding framework presented in Table 1. Content analysis allows making sense of data through systematic coding and comparison (Bogdan & Biklen, 1992). To conduct a content analysis from a deductive approach, first, a draft coding framework was developed based on the standards for mathematical practice defined by CCSSM. Content standards in one strand of each country's standards were coded using the draft framework, through this process the framework was revised and finalized. Then, the author, who is experienced in content standards analysis (Erbilgin, 2014), coded the 7th and 8th grades content standards based on the framework presented in Table 1. Each content standard was reviewed to assess if it includes one or more of the mathematical processes as defined in the framework. Some content standards did not include any mathematical processes. For example, the content standard "7.1.2.2. Convert rational numbers to decimals. (Turkey)" was not assigned any code. Some content standards involved more than 1 mathematical process. For instance, the content standard "N2.c. Discuss examples of direct and inverse proportion and explain the concepts using tables, equations and graphs. (Singapore)" involved 2 processes. It is assigned the code CRO since the standard expects students to discuss and explain mathematical concepts and the standard is also assigned the code UMT since students are expected to use multiple representations to investigate mathematical concepts. When coding a content standard, the remarks and examples are also considered for clarification. When a judgment could not be made for a standard, another mathematics educator knowledgeable about the framework and the content standards was consulted and an agreement was reached.

Patton (2002) suggests that using rigorous data collection and analysis methods increase validity of data. By using a framework based on the related literature, the study aimed to increase the validity of the study. To establish reliability, intra-rater reliability statistics was calculated. One month after completing the coding of each set of curriculum standards, the 7th and 8th grade content standards of Singapore were coded again. The intra-rater reliability score was calculated by dividing the number of same codes by the total number of codes. For instance, for the 7th grade, the initial coding assigned 83 codes to the standards. The re-coding assigned 82 codes to the same standards with 79 of them same as the initial codes. The intra-rater reliability score was calculated to be 95% (79/83). Similarly, the percentage of the same codes between the two separate coding processes was calculated to be 93% for the 8th grade, indicating high reliability (Miles & Huberman, 1994).

Findings

Mathematical Processes in Turkey's Content Standards

Before presenting the findings about content standards, I will briefly present what mathematical processes are emphasized in the introduction part of Turkey' curriculum standards document since it is the basic document outlining the general goals of mathematics education. The document lists problem solving, communication, reasoning, and connections as mathematical processes (MoNE, 2013). These processes are part of the process standards defined by NCTM (2000) and share some common perspectives with m*ake sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others, use appropriate tools strategically, reason abstractly and quantitatively, look for and make use of structure,* and *look for and express regularity in repeated reasoning* mathematical practices defined by CCSSM. The document also includes a section on using information and communication technologies in teaching mathematics effectively. The document advocates a mathematics teaching approach where students are actively engaged in the sense making process, value learning mathematics, and use technology and other representations appropriately (MoNE, 2013).

Table 2 shows the mathematical processes found Turkey's content standards at the 7th and 8th grades. There are 53 content standards at the 7th grade and 48 of them had mathematical processes based on the framework used in this study. There are 54 content standards at the 8th grade and 43 of them had mathematical processes.

Table 2

Mathematical Processes in the Content Standards of Turkey

Mathematical Processes	7th Grade		8th Grade	
	Frequency	Percent*	Frequency	Percent

PS		8	14%	4	6%
RAQ	RA	2	3%	2	3%
	RQ	4	7%	2	3%
CACRO	CA	1	2%	3	5%
	CRO	0	0%	0	0%
M		0	0%	0	0%
UT	UST	22	38%	20	32%
	UMT	2	3%	4	6%
AP	AA	1	2%	4	6%
	AF	0	0%	0	0%
MUS	MUSA	11	19%	15	24%
	MUSC	4	7%	5	8%
ER		3	5%	4	6%
Total		58	100%	63	100%

*Percent values are rounded to the nearest whole number.

Table 2 shows that the most frequent mathematical process integrated into Turkey's 7th and 8th grade content standards is *use appropriate tools strategically* (UT) with majority of them falling into the using single tool category (87%). Since the introduction part of Turkey's mathematics curriculum standards included a section using using technology, I determined the content standards suggesting the use of technology. Out of the 24 UT codes at the 7th grade, 8 of them included using technology for learning mathematics. This number increases to 13 at the 8th grade. One observation about the UT codes was that the content standards typically suggested using technology as a possibility while the other tools are mentioned to be used more strongly. For instance, the remarks for the content standard 7.3.5.2 note using concrete materials as "Constructions made up with congruent cubes and common geometric solids are used." Regarding the use of technology, the same remark notes "Interactional activities with appropriate information and communication technologies might be included." Another observation about the UT codes is that 4 content standards do not specify which tool to use, they rather state using appropriate tools in teaching the related content in a general sense.

According to Table 2, *look for and make use of structure* (MUS) is the second most frequent mathematical process contained in the content standards. Analyzing the structure of shapes, numbers, and expressions, representing them in a different form, and classifying them are involved in about 1/4th of the content standards. *Make sense of problems and persevere in solving them* (PS) was observed in the content standards with 14% at the 7th grade and 6% at the 8th grade. The standards involving PS are all in the form of solving problems related to a mathematical concept such as "7.1.4.7. Solves problems related to direct and inverse proportion." ignoring the other processes involved in problem solving and listed in the framework. The standards assigned the *reason abstractly* (RA) code expected students to construct algebraic representations of real life situations and the standards with *reason quantitatively* (RQ) code expected students to estimate quantities and make sense of quantities in real world contexts. All the content standards that were assigned the *look for and express regularity in repeated reasoning* (ER) code were in the geometry domain and expected students to discover rules or formulas.

Table 2 shows that *model with mathematics* (M) did not appear in the content standards. Similarly, there is not any content standard coded under the *critique the reasoning of others* (CRO) category. The percentages for the *construct viable arguments* (CA) code is also low, indicating that the communication processes are not reflected in the content standards. *Attend to fluency* (AF) is another code that was not observed in Turkey's standards. A related code, a*ttend to accuracy* (AA) was observed in standards expecting students to use accurate labels, terminology, and notations.

Mathematical Processes in Singapore's Content Standards

Singapore's curriculum standards document report that it advocates 3 mathematical processes: Reasoning, communication, and connections; applications and modelling; thinking skilss and heuristics (Curriculum Planning and Development Division, 2012). Reasoning, communication, and connections are common processes with the process standards of NCTM (2000) and shares some aspects of *construct viable arguments and critique the reasoning of others, and reason abstractly and quantitatively* process standards of CCSSM. Application and modelling is about using the modelling process to solve problems within or outside (real-world contexts) mathematics, and it is closely related to the *model with mathematics* standards of CCSSM. It also mentions about *using appropriate tools strategically*. Thinking skills and heuristics highlight mathematical actions such as classifying, comparing, deducting, analyzing, synthesizing, and using a variety of problem solving strategies. The last process involves common perspectives for doing mathematics with the *make sense of problems and persevere in solving them, look for and make use of structure,* and *look for and express regularity in repeated reasoning* standards of CCSSM.

Table 3 shows the mathematical processes found in Singapore's content standards at the 7th and 8th grades. There are 49 content standards (learning experiences) at the 7th grade and all of them had mathematical processes based on the framework used in this study. There are 25 content standards at the 8th grade and all of them had mathematical processes.

Table 3

Mathematical Processes in the Content Standards of Singapore

Mathematical Processes		7th Grade		8th Grade	
		Frequency	Percent*	Frequency	Percent
PS		6	7 %	0	0%
RAQ	RA	7	8%	1	3%
	RQ	10	12%	2	5%
CACRO	CA	5	6%	5	13%
	CRO	10	12%	7	18%
M		2	2%	2	5%

UT	UST	14	17%	9	23%
	UMT	9	11%	5	13%
AP	AA	2	2%	1	3%
	AF	1	1%	0	0%
MUS	MUSA	8	10%	3	8%
	MUSC	5	6%	3	8%
ER		4	5%	1	3%
Total		83	100%	39	100%

*Percent values are rounded to the nearest whole number.

Table 3 shows that the most frequent mathematical process integrated into Singapore's 7th and 8th grade content standards is *use appropriate tools strategically* (UT) with majority of them falling into the using single tool category (62%). Out of the 23 UT codes at the 7th grade, 14 of them included using technology for learning mathematics. This number decreases to 13 at the 8th grade. Regarding the UT category another observation is that Singapore's standards makes specific suggestions for which tools could be used in teaching a concept. For instance, using a clinometer for learning about trigonometric ratios, algebra discs for manipulating algebraic expressions, and specific software names for drawing graphs are suggested in the standards. Table 3 shows that the second most frequently observed mathematical process is *reason abstractly and quantitatively* (RAQ) at the 7th grade and *construct viable arguments and critique the reasoning of others* (CACRO) at the 8th grade. Students are expected to discuss and explain mathematical arguments and contexualize and decontextualize situations when learning mathematics.

At the 7th grade, *model with mathematics* (M) and *attend to precision* (AP) have lower percentage values and at the 8th grade, *make sense of problems and persevere in solving them* (PS), *attend to precision* (AP) and *look for and express regularity in repeated reasoning* (ER) had lower percentage values. The standards involving PS are in the form of solving problems related to a mathematical concept, ignoring the other problem solving processes. However, the standards that were assigned the *model with mathematics* (M) code incorporate other problem solving processes such as understanding the problem context, interpreting a solution in the problem context, and identifying the limitations of the solution.

Comparison of Mathematical Processes for Turkey and Singapore

Figure 1 represents the mathematical processes observed in the content standards of Turkey and Singapore. For both countries, mathematical processes identified in the 7^{th} and 8^{th} grade content standards are combined to make the comparison easier. Tables 2 and 3 show that there are no big differences across the grade levels in terms of use of mathematical processes. Turkey has 107 content standards in total at the 7^{th} and 8^{th} grades and 91 of these standards contained mathematical processes according to the framework used in this study. These 91 standards included 121 processes. Thus, the number of processes are 113% of the content standards (121÷107). For the same grade levels, Singapore has 74 content standards and all of these standards contained mathematical processes. The total number of mathematical processes identified in Singapore's standards is 122. The number of mathematical processes are 165% of the content standards (122÷74). These ratios indicate that Singapore's content standards include more mathematical processes on average than Turkey's standards do.

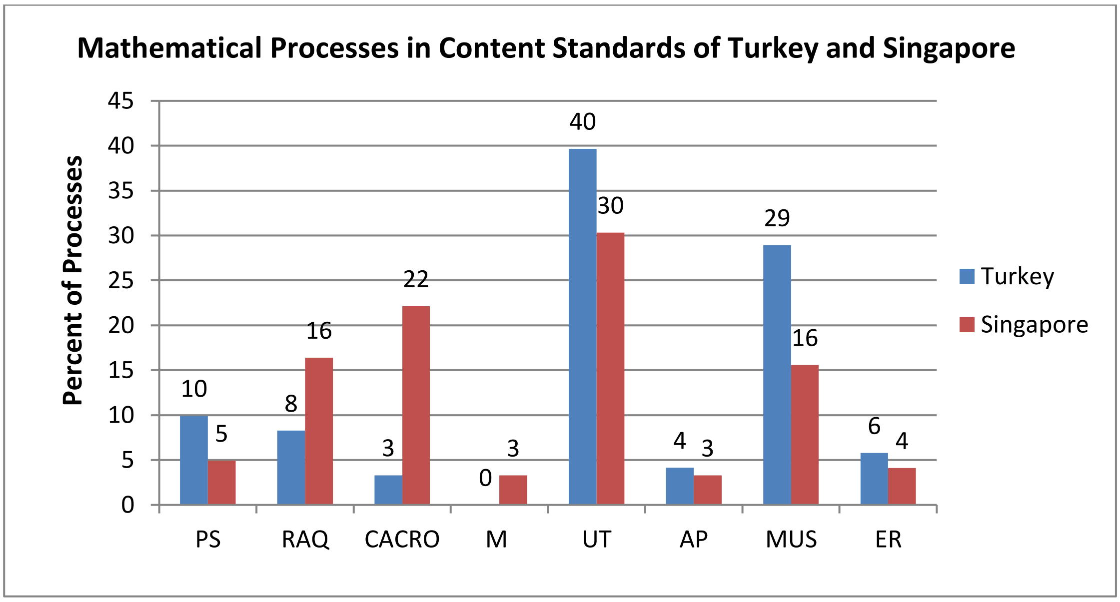

Figure 1. *Mathematical Processes Identified in Content Standards of Turkey And Singapore*

Figure 1 indicates that there are similarities and differences between Turkey's and Singapore's content standards in terms of the highlighted mathematical processes. In both set of standards, *use appropriate tools strategically* (UT) has the highest percentage with 40% for Turkey and 30% for Singapore. Also, in both set of standards, *attend to precision* (AP) and *look for and express regularity in repeated reasoning* (ER) have low percentage values with Turkey's standards having slightly higher values. Turkey's standards do not include any learning expectation that fall into the *model with mathematics* (M) category. Mathematical modelling is closely related to problem solving since it involves solving real-world problems. When *model with mathematics* (M) and *make sense of problems and persevere in solving them* (PS) are combined, both set of standards seem to emphasize problem solving similarly. Nevertheless, Singapore's standards clearly differentiates between problem solving and mathematical modelling as *applications and modelling* is one of its 3 mathematical processes. Additionally, Singapore's standards involve various problem solving processes (e.g., understanding the problem, interpreting the solution) whereas only solving problems is included in Turkey's standards as a problem solving process.

In addition to the diffrence in their approach to modelling, the two set of standards also differ in the *construct viable arguments and critique the reasoning of others* (CACRO) category. CACRO is the second most frequent process in Singapore's standards with 22%. On the other hand, its frequency in Turkey' standards has a value of 3%. Additionally, *critique the reasoning of others* (CRO) component of CACRO was not identified in Turkey's standards. Another difference between the two set of standards is that Singapor's set of standards has a higher percentage of *reason abstractly and quantitatively* (RAQ) caltegory than that of Turkey's. Turkey's set of standards has a higher percentage of *look for and make use of structure* (MUS) than that of Singapore's. Overall, Turkey's content standards focus on UT and MUS with 69% for both categories and put some emphasis on PS and RAQ with 18% in total.

The focus of Singapore's content standards, on the other hand, is more distributed over four processes: UT, CACRO, RAQ, and MUS.

Discussion

This study compared Turkey's and Singapore's mathematics content standards in terms of the highligthed mathematical processes. In the introduction section of their curriculum standards, both countries' documents elaborate some mathematical processes that share common perspectives with the standards for mathematical practice of CCSSM such as *make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others, and reason abstractly and quantitatively*. The current study found that the two countries reflected these processes differently in their content standards. One of the findings is that some of Turkey's content standards did not involve any mathematical process whereas all of Singapore's standards contained at least one process. Another related finding is that *model with mathematics* (M), *critique the reasoning of others* (CRO), and *attend to fluency (AF)* are not identified in Turkey's content standards while all mathematical processes are observed in Singapore's content standards. It is important to note that M, CRO, and AF are mentioned about in the introduction section of Turkey's standards. These two findings indicate that Singapore's standards reflect more variety of mathematical processes with a higher proportion compared to Turkey's standards. Content standards not only determine what topics to be taught in mathematics lessons, but they also guide the development of instructional materials, instructional practices, and assessment (Confrey, 2007; Goertz, 2010; Pang, 2009). Particularly for countries such as Turkey who are struggling to transform to student-centered teaching practices (Güneş & Baki, 2011), more guidence for teachers might be needed. The curriculum standards of Turkey value a student-centered teaching approach as its introduction section implies (MoNE, 2013), but this approach is not widely articulated in the content standards. Future curriculum revisions might consider how to integrate mathematical processes into the content standards to better guide the teachers, textbook writers, teacher educators, and others interested in mathematics education.

The current study provides valuable information about the distribution of the mathematical processes for Turkey and Singapore. The findings revealed that Turkey's standards mainly focus on *use appropriate tools strategically* (UT) and *look for and make use of structure* (MUS). In comparison, the focus of Singapore's content standards is distributed over four processes: *use appropriate tools strategically* (UT), *construct viable arguments and critique the reasoning of others* (CACRO), *reason abstractly and quantitatively* (RAQ), and *look for and make use of structure* (MUS). This finding indicates that both Turkey's and Singapore's content standards promote using representations and tools when learning mathematics and emphasize analyzing the structure of mathematical objects, but compared to Singapore, Turkey's content standards lack a focus on students' reasoning and explaining mathematical arguments. Students' engagement in meaningful mathematical dialogue enhances their own mathematical thinking and also contributes to the common knowledge building process in the classroom (Brendefur & Frykholm, 2000). To promote students' mathematical reasoning and explaining, both CACRO and RAQ could be more explicitly included in the content standards.

Use appropriate tools strategically (UT) has the highest percentage for both Turkey and Singapore. For Turkey 13% of these standards and for Singapore 38% of them fall into the using multiple representations category. Using multiple representations for teaching a mathematical topic and making connections among them enriches students' understanding of mathematical concepts, strengthens the learning process by providing mutual sources of information, and helps to address different types of learners (Brenner, Mayer, Moseley, Brar, Durán, Reed, & Webb, 1997; Porzio, 1999). Regarding the use of tools in Turkey's standards, another observation is that the use of technology in teaching mathematics is presented as an option rather than a strong suggestion like other tools suggested for use. The reason for this choice might be the lack of technological tools in schools. For instance some middle schools do not have a computer lab in Turkey. With the developments in technological tools, technology might be used to increase students' motivation, support conceptual understanding of mathematics, and provide opportunities to do mathematical investigations that cannot be done with paper and pencil (Raines & Clark, 2011; Souter, 2001). Besides, teachers in Turkey possess positive attitudes toward using technology (Cüre & Özdener, 2008; Kılınç, Kılınç, Kaya, Başer, Türküresin, & Kesten, 2016). Students in Turkey should be given more opportunities to use technology to learn mathematics and the content standards might be a starting point for this support.

Turkey's standards include *make sense of problems and persevere in solving them* (PS) and *look for and express regularity in repeated reasoning* (ER) slightly more than that of Singapore's. Both of these processes require higher order thinking skills (Webb, 2007) and it is valuable to highlight them in the content standards. Regarding PS, both countries' content standards contain only "solving problems" aspect of the problem solving process. Singapore, however, integrated various problem solving processes into the content standards that are assigned the *model with mathematics* (M) code. Considering the role of standards in guiding the teachers' practices, including all problem solving processes in content standards might facilitate their practice in classrooms.

Conclusions

One strategy to reform school education have been developing or revising curriculum standards. Curriculum analysis should not be limited to content analysis only, mathematical processes should be also examined. The framework used in this study might be used to study to what extent the processes are integrated into the content standards. Such an analysis will help to allign the mathematical processes advocated by the curriculum developers and the content standards, and assess whether they are indeed reflected in the content standards or not. If the curriculum developers want to convey a coherent message to teachers in the whole curriculum document, then such an analysis is imperative.

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