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On the Excitation of Instability Waves in Plane Poiseuille Flow by a Concentrated Source at the Wall

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إثارة الأمواج غير المتوازنة في سريان بويزيل المستوى بواسطة مصدر مركز على الجدار

ايمن عدنان المعايطة

خلاصة : تبحث هذه الورقة الامواج غير المتزنة بسريان بويزيل المستوى بواسطة مصدر للإضطراب ثنائي الابعاد مركز على احد الجدران وتهدف هذه الورقة إلى دراسة العلاقة ما بين شدة مصدر الإضطراب ومقدار شدة الموجة المثارة لظروف مختلفة من السريان . وقد تم تمثيل مصدر الإضطراب باقتران ديراك ثنائي الأبعاد متناغم مع الزمن . وتم دراسة الإضطرابات الناتجة عن هذا المصدر . ولحل النموذج الناتج تم تطبيق تحولات فورير على معادلات نافير - ستوكس باتجاه السريان . وتم حل المسألة الناتجة باستخدام تحويلات فورير العكسية ونظرية البواقي ومن ثم تم حساب شدة الإضطراب الناتج باستخلاص الأجزاء المسيطرة على الإضطراب المثار . وقد توافقت النتائج النظرية مع القياسات التجريبية المنشورة سابقاً على نفس الموضوع وفسرت بعض هذه القراءات . تقدم النتائج مدى تأثر شدة الإضطراب المثار برقم رينولدز للسريان وترددات مصدر الإضطراب

ABSTRACT: The paper addresses the excitation of instability waves in plane Poiseuille flow by means of a concentrated source at the wall . The aim of this paper is to investigate the relation between the source amplitude and the excited wave amplitude for various flow and source parameters. The source is modeled by a time harmonic Driac function and the generated disturbance is investigated. By applying Fourier transform on the resulting linearized Navier-Stokes equation in the streamwise direction, the problem is resolved using inverse Fourier transform and the residue theorem. The theoretical formulation explains some observations by previous experiments on similar cases of excitations. Moreover, the calculations demonstrate that the disturbance amplitude increases as the excitation frequency decreases for a fixed Reynolds number, while, on the other hand, it increases as Reynolds number decreases for a fixed frequency.

Beyond a certain critical Reynolds number the Plane Poiseuille flow is unstable. Convectively unstable frequency components can be excited by a disturbance located at the wall or inside the mean flow. For example Nishioka, Lida, and Ichikama (1975) excited linearly unstable waves in plane Poiseuille flow by means of a vibrating ribbon placed very near to the wall (0.15, 0.3, and 0.85 mm from the lower wall). Their measurements of the growth rates, the mode shapes of the excited disturbances, along with the critical Reynolds number showed satisfactory agreement with the calculation of Ito (1974) which were based on the linear stability theory. Although Nishioka et al. (1975) measured the amplitude of the disturbance they, however, did not investigate the relation between the amplitude of the excitation source and that of the excited wave. The prediction of the initial disturbance amplitude as compared with that of the excited source amplitude is quite helpful for researchers investigating the excitation of instability waves in plane Poiseuille flows. This is especially important when the nonlinear effect of the excited waves are to be investigated or even avoided. A complete understanding of such a relation and its relevance to flow and source parameters is still lacking for such flow. While the classical stability theory can determine the unstable frequency ranges as well as the phase speed and the growth rates of unstable waves, it comes short of determining the disturbance amplitude.

On the other hand, the relation between source and disturbance amplitudes was investigated for other types of waves in Poiseuille flow. Experimental observations showed that turbulent spots in plane Poiseuille flow can generate growing waves at Reynolds numbers much less than the critical one predicted by the linear theory. Henningson and Alfredsson (1987) investigated such wave packets using a hot-film anemometer. To simulate such a phenomenon Windall (1984) modeled the region of disturbance by a steady traveling delta-function. Li and Windall (1989) extended this work further by representing the turbulent spot as a distribution of increased Reynolds stress. The resulting linearized Naveir-Stokes equations were solved using Fourier Transformation in the plane parallel to the channel walls and a higher-order finite

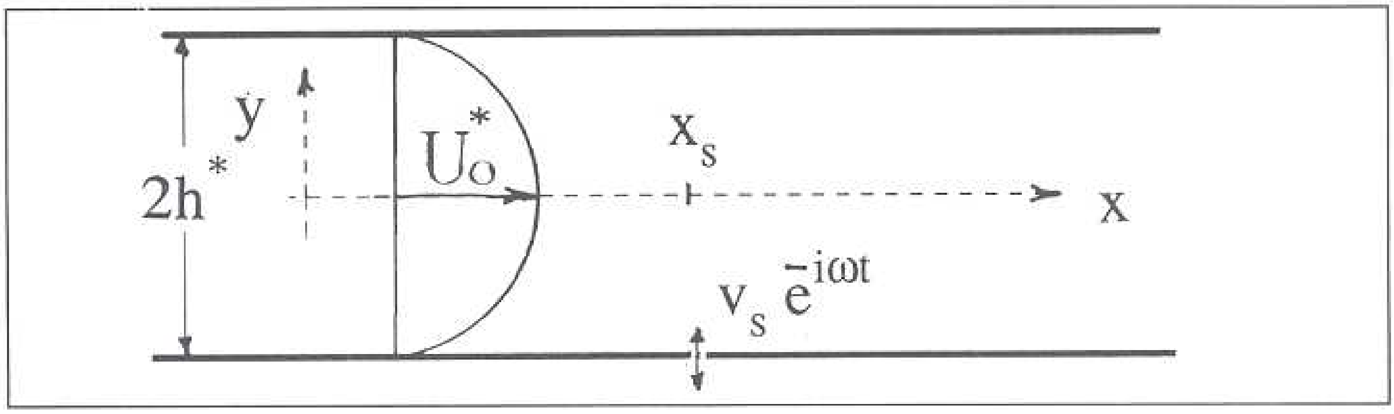

Figure 1. Problem Configuration.

difference scheme across the flow. Hydon and Pedley (1993)and Gogus (1988) theoretically investigated the excitation of Poiseuille flow by oscillating the channel walls. Nonetheless, their aim was to examine the dispersion and the mixing of the flow rather than to excite instability waves in it.

For boundary layer flows Gaster (1965) considered the temporal and spatial initial value problem, i.e. the excitation of instability waves by a localized source. He found that a spatially growing wave is excited downstream from the source; which far downstream corresponds to an eigenmode of the homogeneous instability problem. Tam (1978) provided an almost analytical solution of the problem for the excitation of a two-dimensional free shear layer by an eternal sound field. More recently Michalke and Al-Maaitah (1992) investigated the receptivity of boundary-layer flows over a local velocity profile to a time-harmonic Dirac source. Their calculations demonstrated that velocity profiles inside and close to separation are much more receptive than those away from separation.

In the present paper, the excitation of instability waves in plane Poiseuille flow by means of a two-dimensional concentrated source at the wall is considered. The aim of this paper is to investigate the relation between the source amplitude and the excited wave amplitude for various flow parameters. Section 2 lays out the problem formulation and in section 3 the solution procedure is outlined. Results for various Reynolds numbers and frequencies are presented and discussed in section 4 and some conclusions are then drawn in section 5.

Problem Formulation

Consider a fully developed laminar Poiseuille flow between two parallel plates as shown in figure 1. The parallel non-dimensional mean flow is given by

$$U(y) = (1 - y^2) \tag{2.1}$$

where the non-dimensional velocity U is normalized by the maximum physical velocity at the center U_o^*. Moreover, the lengths (x and y) are normalized by half of the channel width h^*.

When this mean flow is excited by a two-dimensional, harmonic, concentrated source (Dirac source) located at a distance x_s from a reference point, a small disturbance is generated. Here we are not interested in the temporal initial value problem (the switch-on problem), and as such we neglect the transient effects after the excitation. (See Hurre and Monkewitz, 1990). Consequently, our concern is with a two-dimensional disturbance which has non-dimensional velocity components $u_o(x,y,t)$ and $v_o(x,y,t)$ in the x and y directions respectively. Furthermore, the non-dimensional pressure of the disturbance is denoted by $p_o(x,y,t)$. The two-dimensional instability waves have higher growth rates than the three dimensional ones. The excitation of three dimensional instability waves by a two-dimensional source requires a nonlinear subharmonic analysis which is beyond the scope of this paper. Hence we are only interested here in the excitation of the two-dimensional instability waves. The velocity, the length, and time quantities are normalized as explained earlier. For the pressure, the reference density ρ^* is the constant mean flow density. The total flow is then composed of both the mean flow and the disturbance. Substituting the total flow quantities in the Naveir -Stokes (N-S) equations, noting that the mean flow satisfies the N-S equations, and then linearizing for small disturbance the following disturbance equation results:

$$\partial u_o/\partial x_o + \partial v_o/\partial y_o = 0 \tag{2.2}$$

$$\partial u_o/\partial t + U\,\partial u_o/\partial x + v_o\,dU/dy + \partial p_o/\partial x - \{\partial^2 u_o/\partial x^2 + \partial^2 u_o/\partial y^2\}/R = 0 \quad (2.3)$$

$$\partial v_o/\partial t + U\,\partial v_o/\partial x + \partial p_o/\partial y - \{\partial^2 v_o/\partial x^2 + \partial^2 v_o/\partial y^2\}/R = 0 \quad (2.4)$$

where $R = U_o^* h^*/\upsilon^*$ and υ^* is the kinematic viscosity of the mean flow. At the lower wall the boundaty conditions for equations (2.2) - (2.4) are

$$u_o(x, -1, t) = 0 \quad (2.5)$$

$$v_o(x, -1, t) = v_s\,\delta(x - x_s)\,e^{(-i\omega t)} \quad (2.6)$$

where $\delta(x - x_s)$ is the Dirac function and v_s is the amplitude of the excitation source.
The other boundary conditions can be the no-slop and no- penetration condition at the upper wall ($y = 1$). However, since the excited disturbance under consideration is the instability wave beyond the location of the excitation source, then the properties of these waves can be used to define altenative boundary conditions. Consequently, for the purpose of reducing the numerical calculation, it is convenient to define the boundary conditions at the centerline of the flow (i.e. at $y=0$). For even modes the boundary conditions at the centerline are:

$$u_o(x, 0, t) = \partial v_o/\partial y|_{(x,o,t)} = 0 \quad (2.7)$$

On the other hand, for odd modes of instability wave

$$v_o(x, 0, t) = \partial u_o/\partial y|_{(x,o,t)} = 0 \quad (2.8)$$

The use of separate boundary conditions for each mode of instability is valid since we are interested in the unstable modes that are excited by sources of different frequencies. Therefore, the use of conditions (2.7) and (2.8) instead of the no-slip and no- penetration conditions at the upper wall is justified.

Solution Procedure

If it were not for the inhomogeneous boundary condition (2.6), equations (2.2)–(2.4) can be reduced to the conventional stability equations which define the shape of the disturbance and its growth rate but not its amplitude. However, the inhomogeneous term in (2.6) makes the problem similar to a receptivity problem from which the amplitude of the disturbance can be found.

To resolve this problem the $e^{(-i\omega t)}$ term is factored out from all disterbance flow quantities by defining

$$(u_o, v_o, p_o) = [u_1(x,y), v_1(x,y), p_1(x,y)]\,e^{(-i\omega t)}$$

Hence equation (2.2)-(2.4) become:

$$\partial u_1/\partial x + \partial v_1/\partial y = 0 \quad (3.1)$$

$$-i\omega u_1 + U\,\partial u_1/\partial x + v_1\,dU/dy + \partial p_1/\partial x - \{\partial^2 u_1/\partial x^2 + \partial^2 u_1/\partial y^2\}/R = 0 \quad (3.2)$$

$$-i\omega u_1 + U\,\partial v_1/\partial x + \partial p_1/\partial y - \{\partial^2 v_1/\partial x^2 + \partial^2 v_1/\partial y^2\}/R = 0 \quad (3.3)$$

The boundary conditions (2.5) and (2.6) becomes

$$u_1(x, -1) = 0 \quad (3.4)$$

$$v_1(x, -1) = v_s\,\delta(x - x_s) \quad (3.5)$$

At the center line $y = 0$, and for even modes, condition (2.7) becomes

$$u_1(x, 0) = \partial v_1/\partial y|_{(x,o)} = 0 \quad (3.6)$$

For odd modes condition (2.8) results in

$$v_1(x, 0) = \partial u_1/\partial y|_{(x,o)} = 0 \tag{3.7}$$

To be able to apply the Foureir transformation of the distribution with respect to x, it is assumed that all disturbance quantities decay sufficiently rapidly in x. I f an exponentially growing instability wave is generated by the source then $v_1 = 0$ as $x \to -\infty$, but a growing instability wave can exist as $x \to \infty$. In this case we can construct a disturbance field which decays for $|x| \to \infty$, if a suitably chosen solution of the homogeneous problem is superposed. The homogeneous part of the solution has later to be subtracted. Then even in the unstable case the combined disturbance can be assumed to decay for $|x| \to \infty$. By this assumption a Fourier transformation is then applicable. Consequently we define

$$[u(y,k), v(y,k), p(y,k)] = \int_{-\infty}^{\infty} [u_1(x,y), v_1(x,y), p_1(x,y)]\, e^{(-ikx)}dx \tag{3.8}$$

The source velocity at the wall is thus transformed to be

$$V_w = v_s \int_{-\infty}^{\infty} \delta(x-x_s)\, e^{(-ikx)}dx \tag{3.9}$$

Equations (3.1)-(3.3) then become the conventional stability equations

$$ik\, u + dv/du = 0 \tag{3.10}$$

$$i(k\, U - \omega)\, v\, dU/dy + I\, k\, p - [d^2u/dy^2 - k^2 u]/R = 0 \tag{3.11}$$

$$i(k\, U - \omega)\, v\, dU/dy + dp/dy - [d^2v/dy^2 - k^2 v]/R = 0 \tag{3.12}$$

The other boundary conditions then become

$$U(-1,k) = 0 \tag{3.13}$$

For even modes

$$u(0,k) = dv/dy|_{(0,k)} = 0 \tag{3.14}$$

And for odd modes

$$v(0,k) = du/dy|_{(0,k)} = 0 \tag{3.15}$$

Since this is an inhomogeneous problem, the amplitude of v is uniquely defined. That is we can write

$$v(y,k) = c_o\, v_n(y,k) \tag{3.16}$$

where c_o is the amplitude of v and v_n is the normalized mode shape of v.
From (3.5) and (3.9)

$$v(-1,k) = V_w(k) = c_o\, v_n(-1,k) \tag{3.17}$$

so

$$c_o = V_w(k) / v_n(-1,k)$$

Substituting equation (3.17) into (3.16), v_1 can then be found by performing the inverse Fourier transformation of v(y,k) as follows

$$v_1(x,y) = 1/(2\pi) \int_{-\infty}^{\infty} V_w\, v_n(y,k)\, e^{(-ikx)} / v_n(-1,k)\, dk \tag{3.18}$$

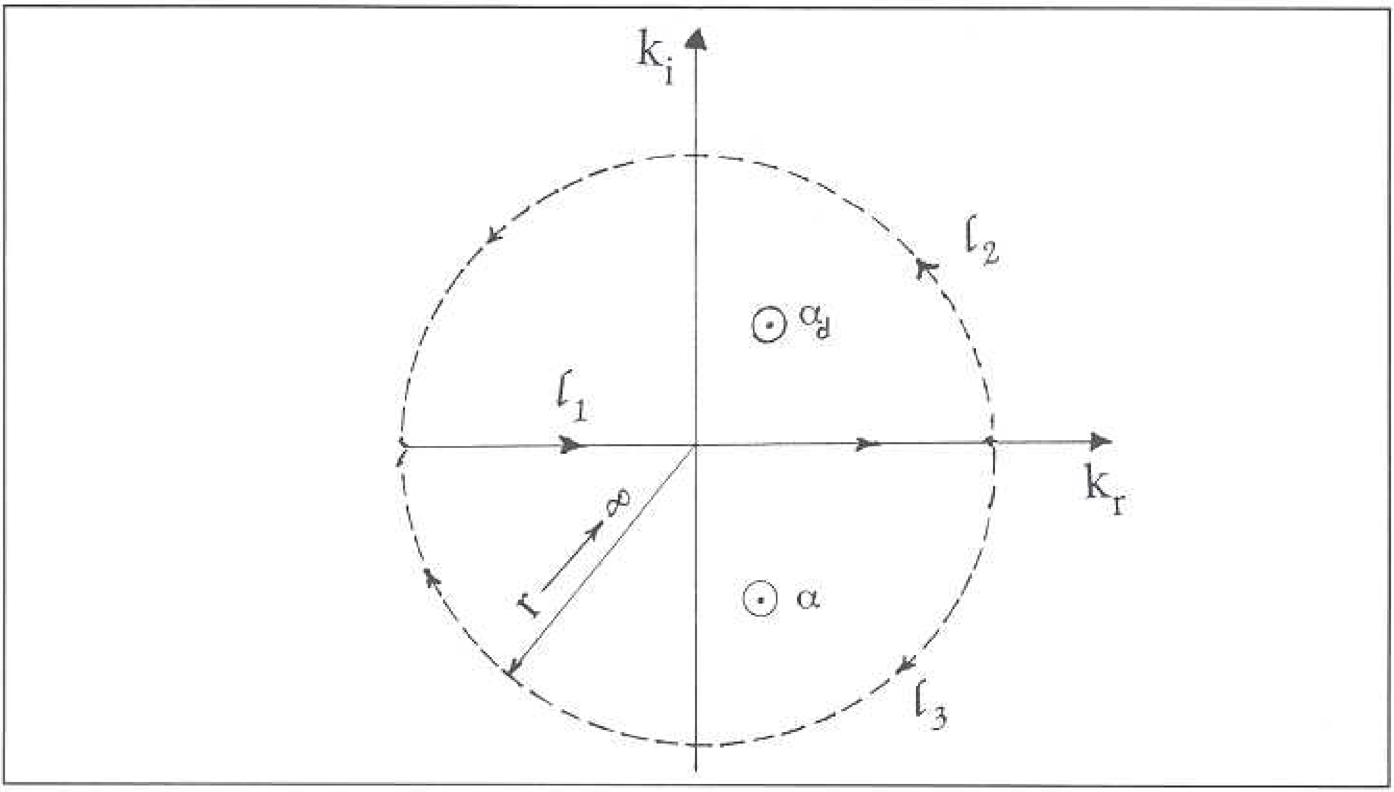

Figure 2. Paths of integration in complex k-plane.

from (3.9) equation (3.18) becomes

$$v_1\,(x,y) = v_s\,/(2\pi)\ {}_{-\infty}\!\int^{\infty}\ v_n(y,k)\ e^{ik(x-xs)}/\ v_n(-1,k)\ dk \qquad (3.19)$$

The evaluation of this Fourier integral is conveniently performed in the complex k-plane. The regular unstable eigenvalues have the property that they move into the upper –k plane for complex frequency $\omega = \omega r + \omega i$, if $\omega i > 0$ is sufficiently large. This is important for the semi-receptivity problem since only the regular eigenvalues that cross the real k axis as ωi goes to zero lead to a causal solution of the receptivity problem and to excited instability waves, as shown in Michalke and AL-Maaitah (1992) and in agreement with the Briggs method [Briggs, 1964]. Hence to perform the only condition required, the integrand must be bounded and analytical for all values of k except at certain poles. Then the residue theorem can be applied as explained with the help of Figure 2. When $(x-x_s) > 0$ the integrand is bounded only in the upper half plane where ki is positive. Hence from the residue theorem the integration over the paths l_1 and l_2 is

$$[\,I\,l_1 + I\,l_2] = i\,2\ Res^u\ H(x-x_s) \qquad (3.20)$$

where Res^u is the residual from the integrand due to any poles of positive ki, and $H(x-x_s)$ is the Heaviside function, and

$$H(x-x_s\,) = \{0 \text{ if } x < x_s\} \text{ and } H(x-x_s\,) = \{1 \text{ if } x > x_s\}$$

Here the Heaviside funciton means that equation (3.20) is valid for $x > x_s$. Similarly for $(\,x-x_s\,) < 0$, the integrand is bounded only on the lower half plane, So performing the integration over l_1 and l_3 yields

$$[\,I\,l_1 + I\,l_3] = -i2\pi Res\ H(x-x_s) \qquad (3.21)$$

Here again (Res) is the residual of the integration resulting from the integrand poles in the lower half-plane. Although the reader might think that the present analysis is valid only upstream of the excitation source, this elusion can be removed as follows. Noting that $H(x-x_s) = 1 - H(x-x_s\,)$ and combining equation (3.20) and (3.21) we obtain

$$2\,\pi\ v_1/v_s = I\,l_1$$

$$= [\, I\, l_3 - I\, l_2 + i\, 2\, \pi\, Res^u \,]\, H(x-x_s) - i\, 2\, \pi\, Res[H(x-x_s)-1] \qquad (3.22)$$

Examining the bracket multiplied by the (Res) term indicates that the complete solution contains an unstable part of the homogeneous problem (-Res) and an unstable part of the inhomogeneous problem, $H(x-x_s)$ Res. The former is only necessary to insure convergence of the Fourier integrals for $x\to\infty$, as was mentioned earlier [see discussion before equation (3.9)]. Consequently, this term has to be subtracted from (3.22). One can also argue physically as follows: The term $\{-H(x_s-x)Res\}$ which is equal to $\{[H(x-x_s)-1]Res\}$ represents an instability wave which exist only upstream of the source. Since all unstable waves can only propagate downstream ($\alpha r> 0$ for $\omega i > 0$) this contribution is physically unrealistic and has to be removed by a suitable solution of the homogeneous equation. As such, the remaining unstable part of the complete solution, which is all that is of interest here, is given by

$$V_{inst} = iH(x-x_s)\, v_s\, Res \qquad (3.23)$$

which indicates that the unstable part exists only down stream of the source position, as was already stated by Gaster (1965) and Hurre and Monkewitz (1990). Consequently performing the integration for the lower k-plane when $x-x_s< 0$ does not restrict the results to be valid only downstream from the excitation source. In fact the integration of equation (3.19) is on the real k-axis and has contributions from upper k-plane (where $x-x_s$ is assumed to be positive) and from the lower k-plane (where $x-x_s$ is assumed to be negative).

The poles of the integrand in (3.22) occur when $v_n(-1,k) = 0$. This happens when k is equal to the eigenvalue of the homogeneous problem α . Spatially amplified disturbances possess a negative imaginary part ki which lie in the lower half-plane of k.

The residue term in equation (3.21), $Res(k=\alpha)$, can be determined if we assume a linear zero of $v_n(-1,k)$ in the neighborhood of α, i.e.

$$v_n(-1,k) = v_n(-1,\alpha) + \partial v_n(-1,k)/\partial k|_{k=\alpha}(k-\alpha) + \ldots..$$

Since $v_n(-1,\alpha) = 0$ then from (3.19) we find

$$Res(k=\alpha) = v_n(y,\alpha)\, e^{i\alpha(x-xs)} / [\,\partial v_n(-1,k)/\partial k|_{k=\alpha}] \qquad (3.24)$$

which is proportional to the eigenfucntion of the homogeneous instability problem. On the other hand, the term Res^u in equation (3.20) results from poles that exist when k equals the eigenvalue of the damped instability wave since they lie in the upper half-plane of k. Consequently, Res^u decays exponentially with x and can be neglected when $(x- x_s)$ is large enough. Moreover, if the radius r goes to infinity then $I\, l_3$ and $I\, l_2$ decay exponentially. This can be noted by inspecting the domains of k and $(x- x_s)$ on which the integral is defined. Furthermore, $I\, l_1$ becomes the integral (3.19) as r goes to infinity. Consequently, from equation (3.22) the Fourier integral (3.19) is composed of the unstable exponentially growing contribution due to excitation, with the factor $[H(x-x_s) -1]$. The decay of Res^u term with x might explain the observations reported by Nishioka *et al.*(1975). They noted that "some distance from the ribbon was required for the disturbance to establish a switch for did not change downstream". This can be attributed to the fact that the excitation source also excites decaying modes of instability as indicated by equation (3.10) and (3.21). These modes then damp out as they travel leaving only the unstable mode in the flow. As explained earlier the term multiplied by $H(x-x_s)$ is part of the solution of the in-homogeneous problem, while the term multiplied by (-1) is a solution of the homogeneous problem which is independent of the excitation. This latter term is only necessary to ensure boundeness of the Fourier integral as $x\to\infty$. Hence we have to subtract this term and can neglect the near-field influence of Res^u, $I\, l_3$ and $I\, l_2$ in order to obtain the dominant unstable part v_{inst}, excited by the source, of the inhomogeneous solution. Hence from (3.23)

$$V_{inst}(x,y)/ v_s = H(x-x_s)\, A\, v_n(y,\alpha)\, e^{i\alpha(x-xs)} \qquad (3.25)$$

With the excited amplitude

$$A = i/\{\partial v_n(-1,k)/\partial k|_{k=\alpha}\} \qquad (3.26)$$

And as such we define $A_v = |\, A\,|$. It should be noted that the amplitude A ,of the disturbance depends on the normalization of the eigenfunction. Physically this means that the measured amplitude depends on the location at which the measuring instrument is placed since the disturbance varies with y. For equation (3.26) v_n is normalized such that its maximum value at y = 0 is unity. This means that A_v is the amplitude of v component of the disturbance at y = 0. Another way of normalization can be by making the maximum value of $u1(y,\alpha)$ to be unity. Hence w define

$$A_u = A_v \mid u_{max} / v_n(0,\alpha) \mid \tag{3.27}$$

where the u_{max} is the maximum value of $u(y,\alpha)$. Furthermore, an other form of the amplitude, which takes into account the growth rate of the disturbance, is A defined as the amplitude of the disturbance as it travels a distance of one wavelength down stream of the source. Hence define

$$A_\lambda = A_v \, e^{-2\pi\alpha i/\alpha r} \tag{3.28}$$

where αi and αr are the real and imaginary parts of α, respectively. It is clear from equation (3.24) that the disturbance amplitude is proportional to that of the source amplitude that is expected from the present linear theory.

Thus for a given excitation source frequency, the amplitude of the disturbance can be found by calculating $\partial v_n(-1,k)/\partial k|_{k=\alpha}$. The value of $v_n(y,\alpha)$ is calculated from equations (3.10)-(3.12) and the homogeneous boundary conditions using a high order finite difference method. Consequently, the excited instability wave given by equation (3.24) is completely computed.

Numerical Results

For certain R and ω, the eigenvalue α and the eigenfucntion $v(y,\alpha)$ can be found by solving the stability equations (3.10)-(3.12) using the homogeneous boundary conditions (3.13)-(3.15) and for a homogeneous form of (3.17). In solving these equations a finite difference method is utilized using the variable step-size finite-difference code PASAV3, (see Pereyra 1976). To utilize the code equations (3.10)-(3.12) are re-written as a system of four first-order complex differential equations (or 8 real ones). Coupled with a Newton-Raphson procedure, the eignevalue problem is solved using 101 points across the y domain (from −1 to 0). This method is quite efficient and was previously used to solve the stability problems of compressible and incompressible boundary layer flows, (e.g. Al-Maaitah *et al.* 1990). The calculated eigenvalue and eigenfucntions totally agree with the calculation of Ito (1974). To verify the accuracy of the present code, its results were also compared with the temporal calculation of Thomas (1953). In doing so we fixed α to be real and searched for complex ω. The calculated eigenvalues agree up to 5 digits. Since the present paper is concerned with spatial propagation of the disturbance, the temporal analysis was performed only for the sake of comparison. The absolute values of the eigenfunctions for spatial modes are shown in figure 3. Here the eigenfunctions are normalized such that v(0)=1.

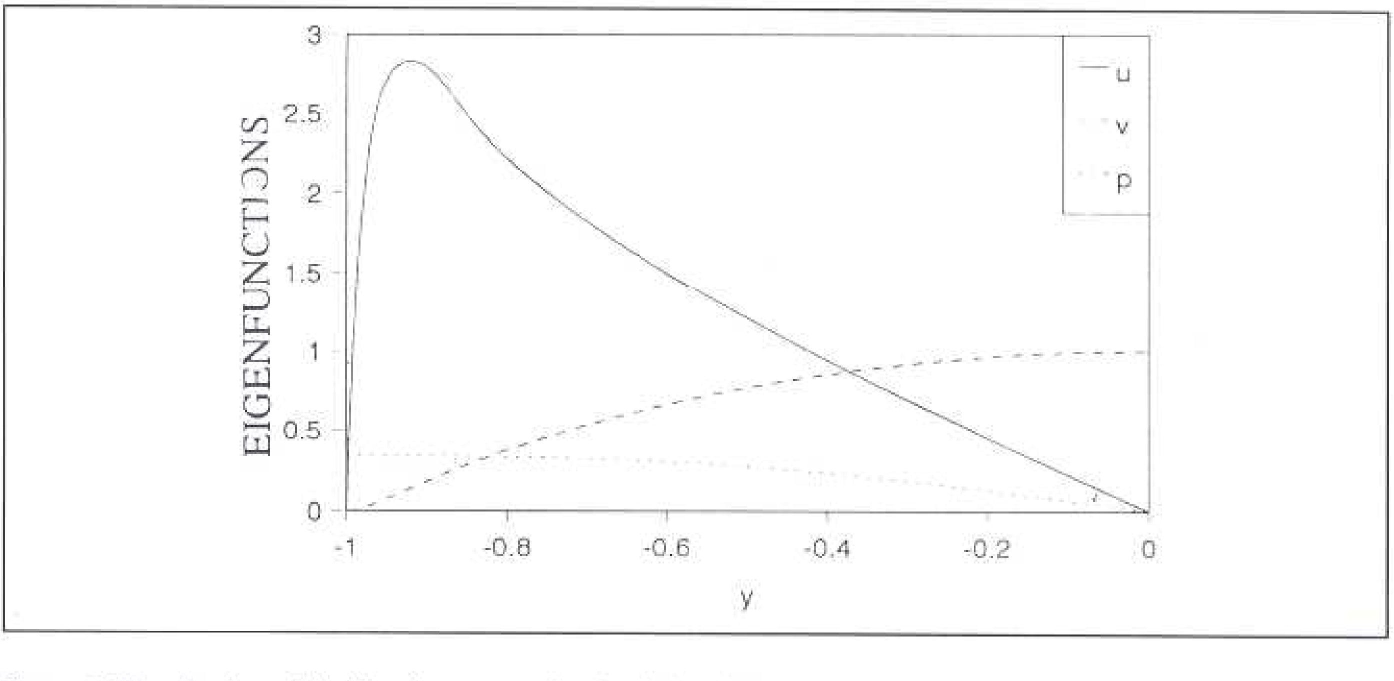

Figure 3. Eigenfunctions of the disturbance normalized such that v(0) is unity. Here R= 4000, ω = 0.013 and a = 0.79 - i 0.031.

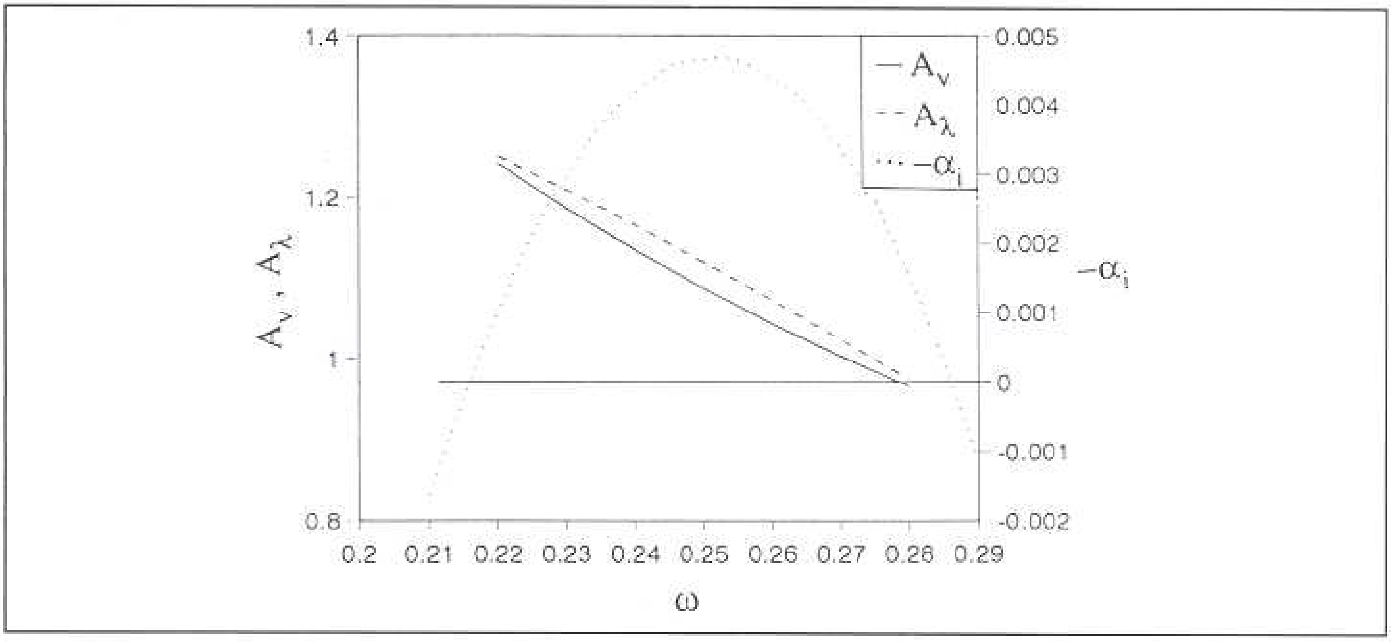

Figure 4. The variation of the growth rates, A_v and A_λ with ω for R = 7000.

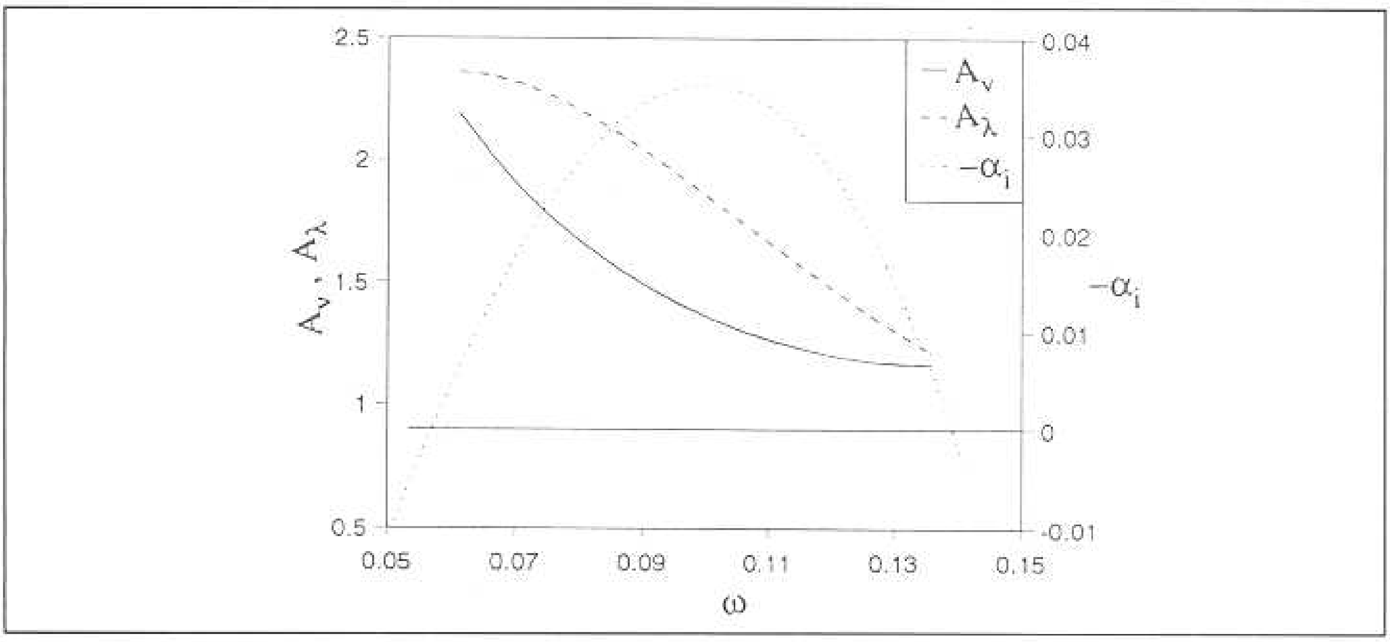

Figure 5. The variation of the growth rates, A_v and A_λ with ω for R = 80000.

In order to calculate the amplitude A_v of the excited instability wave according to (3.25), the derivative of v_n with respect to k is calculated using central differences as

$$\partial v_n(-1,k)/\partial k|_{k=\alpha}(k-\alpha) = \{v_n(-1, \alpha + \Delta k) - v_n(-1, \alpha - \Delta k)\}/(2\Delta k) \qquad (4.1)$$

It was checked that the complex value was independent of the phase of Δk for sufficiently small values of Δk. In the present results $\Delta k = (1+i)\times 10^{-5}$. Hence A_v is calculated from equation (3.25) and (3.26). Consequently A_v and A_λ are calculated from equations (3.27) and (3.28).

Although the present theory is valid for all unstable modes, we present results for even modes since odd modes are stable for plane Poiseuille flow. When R= 7000, Figure 4 shows the variation of the growth rate with the non-dimensional frequency ω. the figure also demonstrates the variation of the amplitude of the disturbance v component at y=0 (A_v), and the amplitude of the v component at y=0 as the disturbance travels one wave length down-stream from the source (A_λ). It is evident that both A_v and A_λ decrease as ω increases for a fixed R. Around the lower branch of stability A_v approaches

A_λ for obviuos reasons where the disturbance amplitude is nearly 1.25 Vs. On the other hand, A_v and A_λ are less than unity around the upper branch. Since the Reynolds number in the results presented in Figure 4 is around the critical one, the growth rates are small, consequently the values of A_λ are close to those of A_v. When R is as large as 80000, then A_v and A_λ are higher than unity for the whole band of unstable frequencies as shown in Figure 5. In fact A_v and A_λ are more than 2 around the lower branch of stability. Furthermore, A_λ is significantly greater than A_v for this Reynolds number due to the relatively large growth rates. Nonetheless, both A_v and A_λ continue to decrease as ω increases.

While A_v and A_λ represent the magnitude of the v component of the disturbance, A_u represents the maximum amplitude of the u component of the disturbance. The relation between A_v and A_λ can be determined from conventional linear instability theory. Figure 6 illustrates the variation of A_u / A_v with ω when R = 14000. Around the lower branch of instability A_u / A_v decreases slightly and then levels off as ω increases. It is worth noticing that A_u / A_v is not very sensitive to ω and A_u is nearly three times A_v.

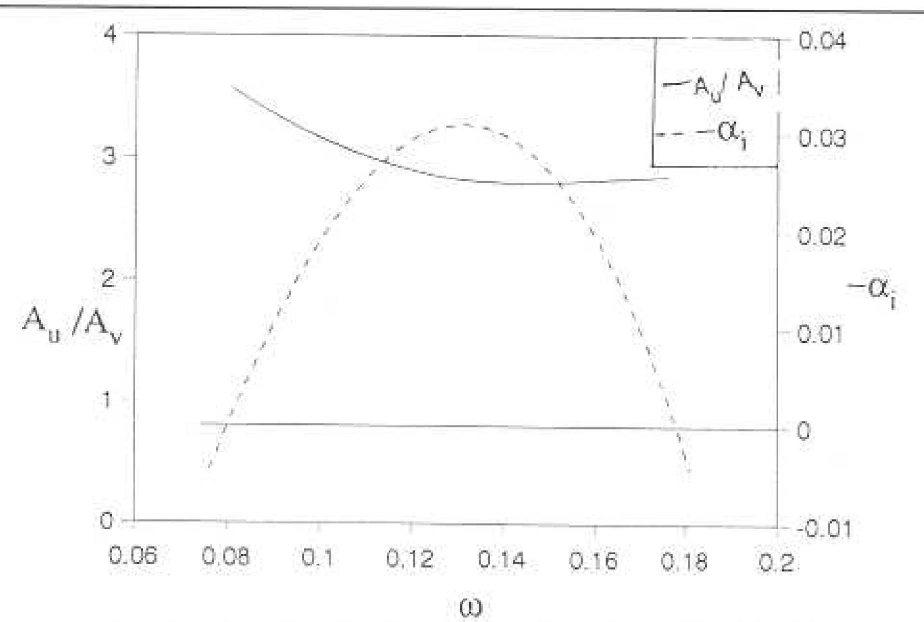

Figure 6. The variation of the growth rates and A_u / A_v with ω for R = 14000.

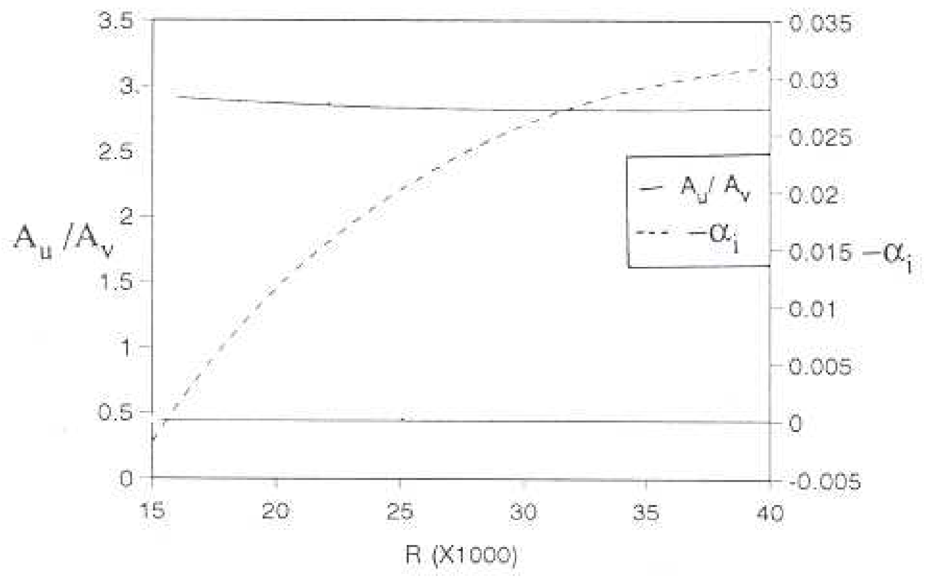

Figure 7. The variation of the growth rates and A_u / A_v with R when ω = 0.131.

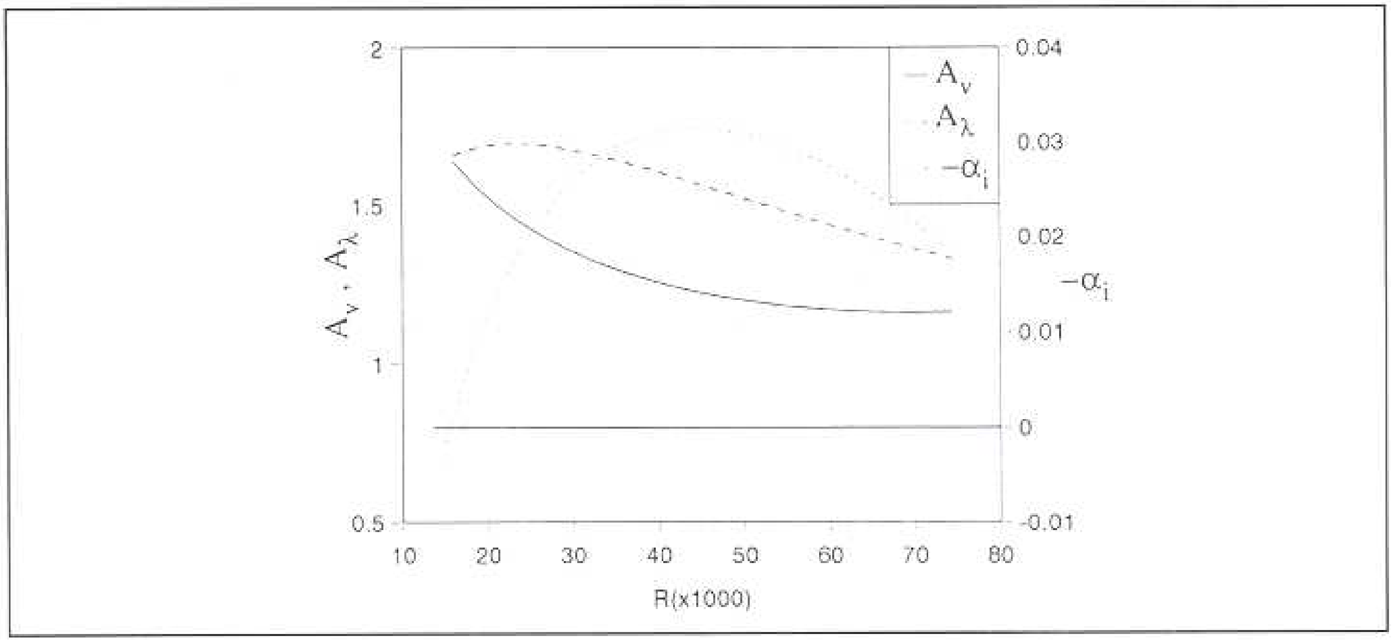

Figure 8. The variation of the growth rates, A_v and A_λ with R when $\omega = 0.131$.

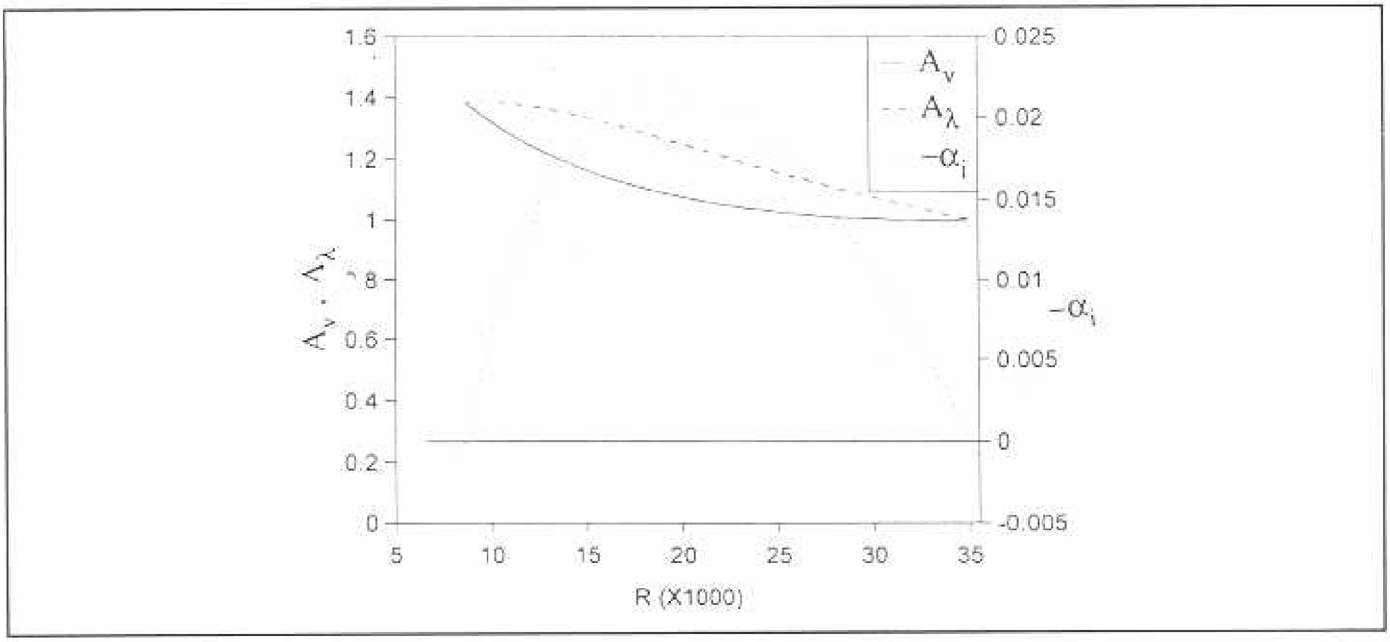

Figure 9. The variation of the growth rates, A_v and A_λ with R when $\omega = 0.186$.

The previously discussed results were investigating the influence of ω, which is a parameter of the excitation source. Next the influence of the Reynolds number, a flow parameter, is investigated. Figure 7 shows the variation of A_u / A_v with R for ω =0.131 demonstrating that this ratio has slow variation with R and its value is around 3. Figure 8 illustrates the variation of A_v , A_λ and the growth rate with R for ω =0.131. It appears that while A_v decreases monotonically with R, A_λ increases slightly around the lower branch of stability and then decreases slightly around the lower branch of stability and then decreases continuously as R increases. In comparison with its variation with the frequency the disturbance amplitude is not as sensitive to changes in R as its sensitivity to changes in ω. When ω =0.186 the variation of A_v , A_λ and the growth rate with R is demonstrated in Figure 9. The same behavior of A_v and A_λ with R can be noted. Although figures 8 and 9 show that A_v , A_λ decrease as R increases for fixed frequency, this is not the case when the maximum values of A_v and A_λ are considered over the entire range of unstable frequency. By investigating Figures 10.a-10.d (which show the variation of A_v and A_λ with ω for R=9000, 16000, 25000, and 40000) and recalling Figures 3 and 4, it is obvious that the maximum disturbance amplitude over the unstable frequency band increases as R increases. On the contrary, figures 11.a-11.b, along with figures 8 and 9, demonstrate that the maximum disturbance amplitude decrease

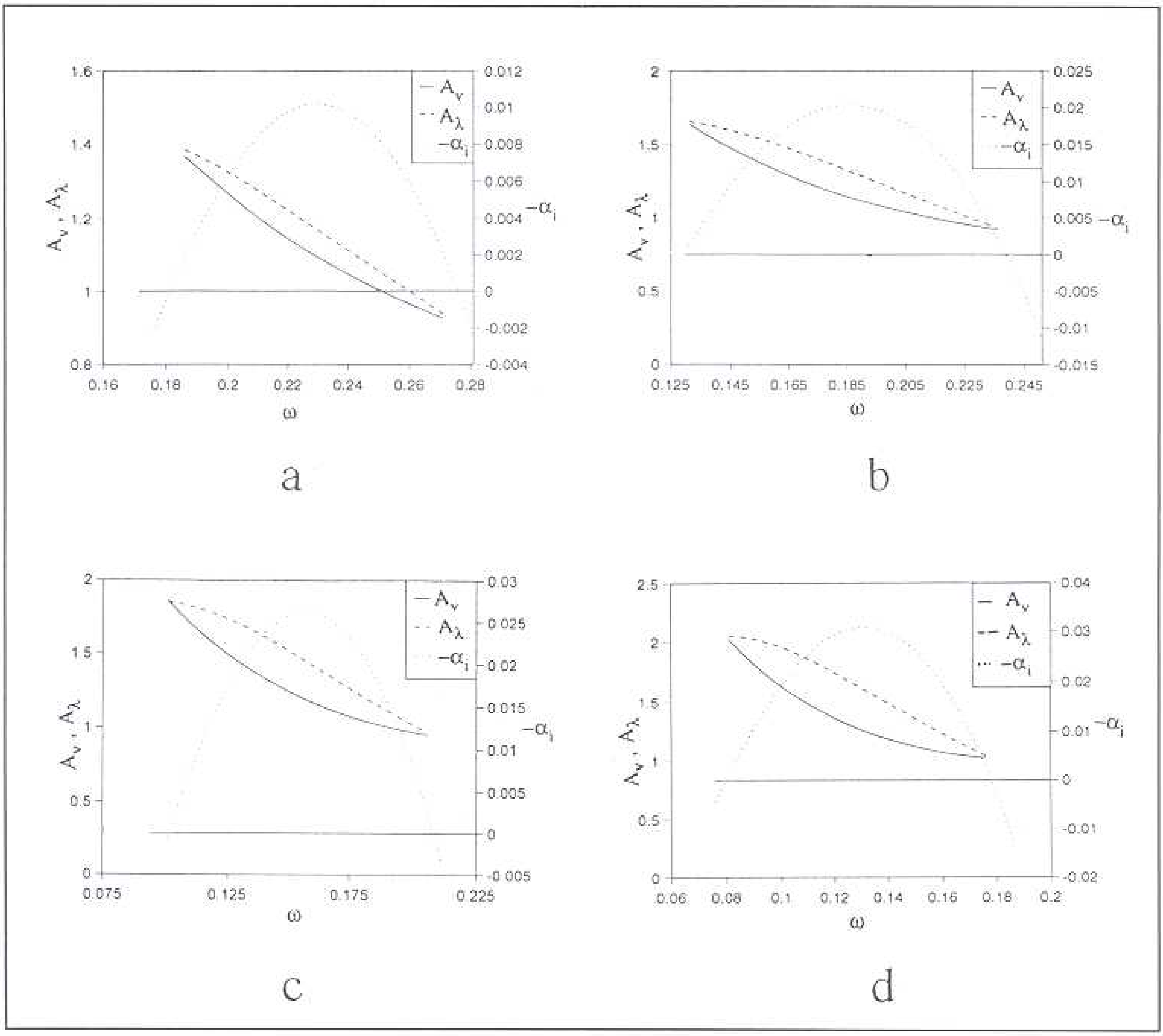

Figure 10. The variation of the growth rates, A_v and A_λ with ω for various Reynolds numbers; a- R = 9000, b- R = 16000, c- R = 25000, d- R = 40000.

as the frequency increases. An interesting case is demonstrated in Figure 11.b where ω is as low as 0.061 for high Reynolds numbers. Due to the large growth rates A_λ has a local maximal after which it decreases as R increases. Although for intermediate ranges of Reynolds number A_v is a strictly decreasing function, the results for the conditions of Figure 11.b show that A_v has a local minimum after which it increases as R increas.

Conclusions

Even though, no experimental results are available to compare with, the present theory explains the observation of Nishioka *et al.* (1975) regarding the change of the disturbance behavior in the neighborhood of the excitation source. The results show that the disturbance amplitude is more sensitive to excitation source parameters than to flow parameters. Although the disturbance amplitude decreases as R increases for a fixed frequency, the maximum amplitude over the results demonstrate that the flow is more receptive to wall excitation when flow and source parameters correspond to a disturbance parameters which are close to the lower branch of instability. The maximum amplitude of the v component of the disturbance velocity is around twice the amplitude of the source while that of the u component is around 7 times the excitation source amplitude. Hence it can be concluded that unlike the case of Blasius flow, the described procedure of excitation can produce a distribution in plane Poiseuille flow with higher amplitudes in comparison.

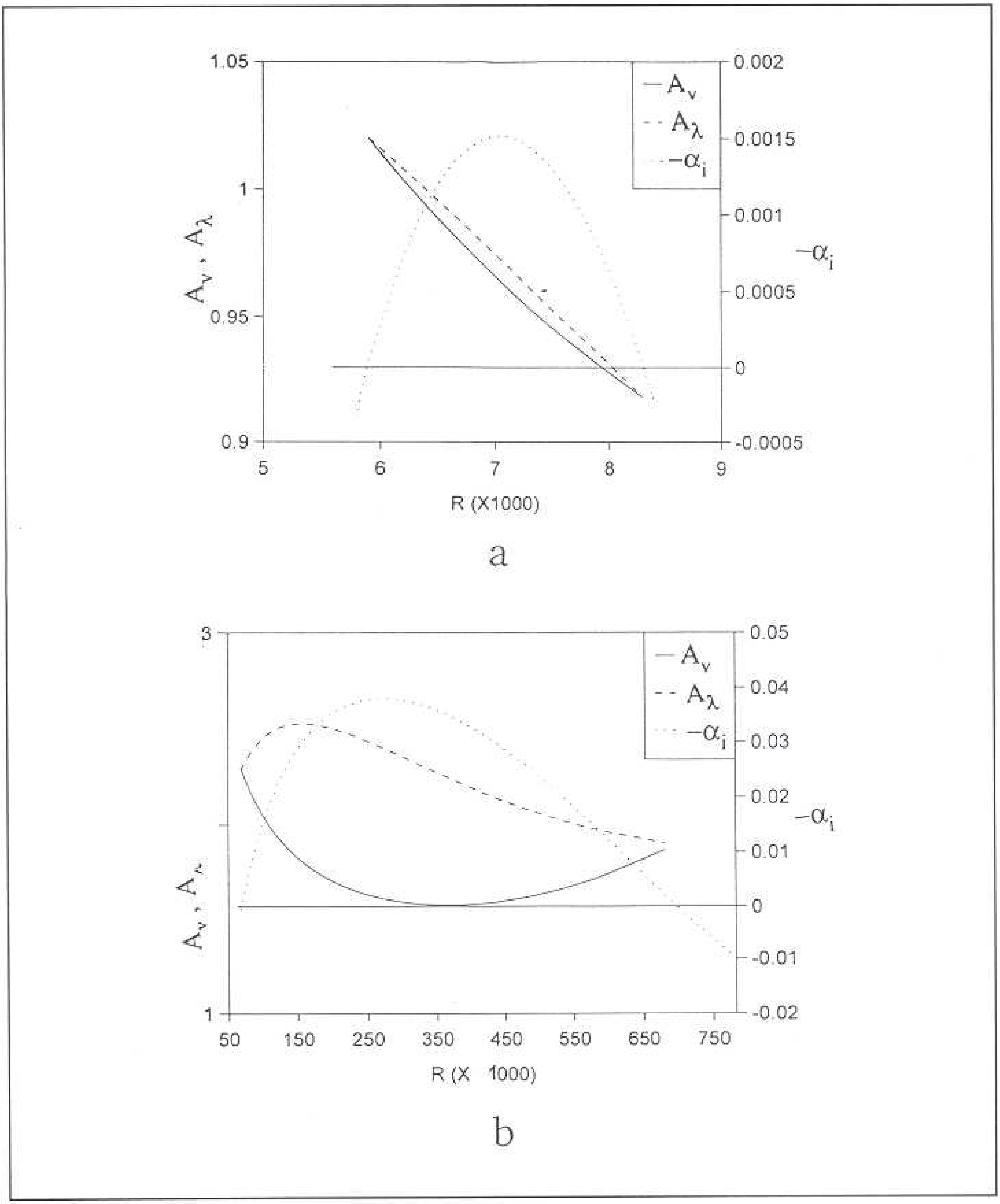

Figure 11. The variation of the growth rates, A_v and A_λ with R for various ω; a- ω = 0.28, b- ω = 0.061.

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