SQU Journal for Science, 2018, 23(1), 8-18 DOI: http://dx.doi.org/10.24200/squjs.vol23iss1pp8-18
Sultan Qaboos University

The Mathematical Modelling of a Fixed Source of Dust

Khaled S.M. Al-Mashrafi

Department of Human Resources Development, General Directorate of Education in Eastern Region, Ministry of Education, Sur, P.O. Box 998, P.C. 411, Sultanate of Oman. Email: khaled2014om@gmail.com

ABSTRACT: A mathematical model for the diffusion of dust particles emitted from a fixed source is investigated using the atmospheric diffusion equation. This model poses an initial boundary value problem with a second order linear partial differential equation. The steady state case of this problem when the uniform source is situated at ground level was examined by Sharan *et al*. [1]. The solution of the unsteady case in closed form for a time dependent source is derived. Two special cases, in which the source function of time is explicitly given and special values of the diffusion parameters are taken, are examined in detail. In the case when diffusion is present only in the vertical direction, it is shown that for small times, the particles spread with a front that travels with the speed of the wind. When diffusion is present only in the direction of the wind, there is no discontinuity front and the particles diffuse slowly into the direction of the wind. The solutions for the special cases considered are examined for large values of time. It is found that the solution approaches that of the corresponding steady state solution of the equation.

Keywords: Atmospheric diffusion equation, Vertical diffusion, Horizontal diffusion, Wind speed.

النموذج الرياضي لانتشار ذرات الغبار التي تنبعث من مصدر ثابت

خالد سليمان المشرفي

الملخص: تمت دراسة النموذج الرياضي لانتشار جـُزئيات الغـُبار المنبعثة من مصدر ثابت باستخدام معادلة الانتشار الجوية. يتطلب هذا النموذج دراسة معادلة تفاضلية جزئية خطية من الدرجة الثانية بشروط ابتدائية حدودية. اختبرت هذه الحالة مسبقاً من قبل شاران ورفاقه [1] في الحالة الخاصة التي يكون فيها مصدر الغبار متمركزا على سطح الأرض ومستقلا عن الزمن. ووجد حل ظاهري لجميع المصادر التي ترتبط بالزمن. تم في هذا البحث اختبار حالتين خاصتين بالتفصيل عندما يكون المصدر دالة في الزمن، مع أخذ حالات خاصة لمعاملات الانتشار. عند وجود إنتشارعمودي فقط ولزمن قصير، تم اثبات أن ذرات الغبار تنتشر بالمقدمة وبنفس سرعة الريح. أما في حالة إنتشار أفقي فقط وبنفس إتجاه الريح، تم ملاحظة أن الغبار ينتشر ببطء في اتجاه الريح. كذلك تم اختبار حلول الحالات الخاصة التي تم افتراضها عندما يكون الزمن طويلا. وتم اثبات أن الحل يقترب من الحل الثابت لتلك المعادلـة.

الكلمات المفتاحية: معادلة الانتشار الجوية، انتشار رأسي، إنتشار أفقي، سرعة الريح.

1. Introduction

The mathematical model of particles emitted from a fixed source has been investigated [1-11]. The study of transport of such particles by wind in the atmosphere is important because it causes problems to the environment [12-15]. Most industrial establishments have factories with chimneys through which the fumes escape into the atmosphere outside the factory. These fumes diffuse into the surroundings causing pollution and forming a health hazard. In arid lands, as in Oman, strong winds carry dust from the ground and transport it, which can cause damage to roads and also dirt in houses [16-17].

The diffusion of particles emitted from a source in the atmosphere is given by the atmospheric diffusion equation [18].

$$\frac{\partial C}{\partial t^*} + \mathbf{u}.\nabla C = \nabla.(\underline{\mathbf{D}}:\nabla C) - \mathbf{w}.\nabla C, \tag{1}$$

where C is the concentration of the particles after time t^*, $\mathbf{u}$ is the local velocity of the particles, $\mathbf{w}$ is the settling velocity, and $\underline{\mathbf{D}}$ is the stress tensor given in a Cartesian coordinates system by

CC BY ND

$$\underline{\mathbf{D}} = \begin{pmatrix} D_{x^*x^*} & D_{x^*y^*} & D_{x^*z^*} \\ D_{y^*x^*} & D_{y^*y^*} & D_{y^*z^*} \\ D_{z^*x^*} & D_{z^*y^*} & D_{z^*z^*} \end{pmatrix},$$

where the axis Oz^* is vertically upwards and the axes Ox^* and Oy^* are horizontal.

The analytical study by Sharan *et al.* [1] investigated a steady-state model for low wind speeds where the gravitational force is negligible. The air stream moved with a uniform velocity U in the $x-$direction. They assumed the presence of diffusion components in three coordinate directions, all of which being linearly proportional to the distance along the wind. They concluded that their result is in reasonable agreement with experimental observations. In this paper, we extend the steady state model by Sharan *et al*. [1] to include the time variation. We assume that the wind speed is low, the diffusion varies linearly with distance along the wind direction, and gravity is ignored. In section 2, the model of the system is formulated. In section 3, the solution is calculated and examined in detail for some special cases of the diffusion parameters and the time dependence of the source. A general discussion of the solution is also presented in this section. Some concluding remarks are made in section 4.

2. Formulation of the model

The diffusion of dust particles emitted from a fixed source situated on/or above ground level in the atmosphere is governed by the atmospheric diffusion equation (1). This equation can be written in a Cartesian system of coordinates $O\left(x^*, y^*, z^*\right)$ in the absence of the settling velocity as

$$\frac{\partial \mathrm{C}}{\partial t^*} + \mathbf{u}.\nabla \mathrm{C} = \frac{\partial}{\partial x^*}\left(D_{x^*x^*}\frac{\partial \mathrm{C}}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(D_{y^*y^*}\frac{\partial \mathrm{C}}{\partial y^*}\right) + \frac{\partial}{\partial z^*}\left(D_{z^*z^*}\frac{\partial \mathrm{C}}{\partial z^*}\right), \tag{2}$$

where $D_{x^*x^*}$, $D_{y^*y^*}$, $D_{z^*z^*}$ are the components of the stress tensor in x^*, y^*, z^* directions, respectively.

Assume that the direction of the wind speed is in the x^* – axis and the velocity of dust particles is given by

$$\mathbf{u} = (U, 0, 0) + (u, v, w), \tag{3}$$

where (u, v, w) is the velocity of dust particles relative to the local wind speed and $(U, 0, 0)$ is the wind speed.

If we also assume that the components u, v, w of the dust particles' velocity are very small in comparison with U, and the variations of concentration in all directions are similar, then the advection term in (2) reduces to

$$\mathbf{u}.\nabla \mathrm{C} \sim U\frac{\partial \mathrm{C}}{\partial x^*}. \tag{4}$$

Assume further that all three components of the diffusion are linear in x^*:

$$D_{x^*x^*} = \alpha U x^*, \quad D_{y^*y^*} = \beta U x^*, \quad D_{z^*z^*} = \gamma U x^*, \tag{5}$$

in which α, β, γ are positive constants.

Using equations (4) - (5), write equation (2) as

$$\frac{\partial C}{\partial t^*} + U\frac{\partial C}{\partial x^*} = \frac{\partial}{\partial x^*}\left(\alpha U x^*\frac{\partial C}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(\beta U x^*\frac{\partial C}{\partial y^*}\right) + \frac{\partial}{\partial z^*}\left(\gamma U x^*\frac{\partial C}{\partial z^*}\right), \tag{6}$$

which represents the linearized three-dimensional unsteady atmospheric diffusion equation in the absence of gravitational forces. This equation is solved subject to the following initial and boundary conditions

$$C(x^*, y^*, z^*, t^*) \to 0 \quad \text{as} \quad x^*, \left|y^*\right|, z^* \to \infty, \tag{7}$$

$$C(x^*, y^*, z^*, 0) = 0, \tag{8}$$

$$C(0, y^*, z^*, t^*) = \frac{\overline{Q}}{U}\,\delta(y^*)\,\delta(z^* - h^*)\,f(t^*) \quad \text{with} \quad f(0) = 0\ ; \quad t^* > 0\ , \tag{9}$$

$$\frac{\partial C(x^*, y^*, 0, t^*)}{\partial z^*} = 0, \tag{10}$$

where the dust particles emanate from a fixed source at $(0, 0, h^*)$ has a strength $\overline{Q}$ for all $t^* > 0$, $h^* \geq 0$, and $\delta(s)$ is Dirac's delta function. It is clear from the condition (9) that the source depends on time t^* and its strength depends on the function $f(t^*)$.

The equation (6) and the relevant conditions (7) - (10) can be written in the following dimensionless form

$$x\frac{\partial^2 C}{\partial x^2}+x\frac{\partial^2 C}{\partial y^2}+x\frac{\partial^2 C}{\partial z^2}+a\frac{\partial C}{\partial x}-\frac{\partial C}{\partial t}=0, \tag{11}$$

$$C(x,y,z,0)=0\ ; \tag{12}$$

$$C(x,y,z,t)\to 0 \quad ; \quad x,|y|,z\to\infty, \tag{13}$$

$$C(0,y,z,t)=Q\ \delta(y)\delta(z-h)f(t)\ ,\ f(0)=0, \quad t>0\ , \tag{14}$$

$$\frac{\partial C(x,y,0,t)}{\partial z}=0, \tag{15}$$

where

$$\left.\begin{array}{c} t=\alpha U t^{*} \quad ,x=x^{*}\ ,\ y=\sqrt{\dfrac{\alpha}{\beta}}\,y^{*}\ ,\ z=\sqrt{\dfrac{\alpha}{\gamma}}\,z^{*}\ ,\ a=\dfrac{\alpha-1}{\alpha} \quad ;\alpha,\beta,\gamma\neq 0, \\ h=\sqrt{\dfrac{\alpha}{\gamma}}\,h^{*} \quad , \quad Q=\dfrac{\overline{Q}}{U} \end{array}\right\}. \tag{16}$$

The system (11) - (15) was solved by integral transform methods to obtain the solution for the concentration in the (x,y,z) plane at any time in closed form for a source of general time dependence [10-11]. The solution showed that, as well as the position of the source in the vertical direction, the diffusion parameters α,β,γ play an important role in the spread of the dust particles in the atmosphere. In this paper, we investigate the influence of diffusion parameters in all directions by studying the following two special cases: (1) vertical diffusion $\alpha=\beta=0,\gamma\neq 0$ and (2) longitudinal diffusion $\beta=\gamma=0,\alpha\neq 0$.

3. Analytical solutions for the model

3.1. Case (1): Solution in the case of vertical diffusion ($\alpha=\beta=0,\ \gamma\neq 0$)

In this case, vertical diffusion is present and both longitudinal and latitudinal diffusions are absent. The solution here is given by

$$C(x,z,t)=\frac{Q\ f(t-x)H(t-x)}{2\sqrt{2\pi}\,x}\left[\exp\left\{\frac{-(h+z)^2}{2x^2}\right\}+\exp\left\{\frac{-(h-z)^2}{2x^2}\right\}\right], \tag{17}$$

where $H(s)$ is the Heaviside function, x and Q are defined in (16), and t, z and h are redefined as

$$t=Ut^{*}\ ,\ z=\frac{z^{*}}{\sqrt{\gamma}}\ ,\ h=\frac{h^{*}}{\sqrt{\gamma}}. \tag{18}$$

The solution (17) specifies the concentration at every point (x,z,t) of the domain. It is clear that the time dependence appears only in the amplitude of the concentration and is absent in the exponential dependence. Moreover, the presence of the Heaviside unit function in the amplitude of the solution represents the discontinuity in the solution across the line $x=t$ in the (x,t) plane. The solution (17) is illustrated by two examples of the source $f(t)$, which are: (i) Heaviside function $H(t)$, and (ii) exponential function $1-e^{-\lambda t},\lambda>0$. The aim for choosing these specific examples is to examine the effect of the strength of the source as time varies.

$$(i)\quad f(t)=H(t)=\begin{cases}1 & ,\quad t>0\\ 0 & ,\quad t<0\end{cases}$$

The solution (17) reduces to

$$C(x,z,t)=\begin{cases}\dfrac{Q}{2\sqrt{2\pi}\,x}\left[\exp\left\{\dfrac{-(h+z)^2}{2x^2}\right\}+\exp\left\{\dfrac{-(h-z)^2}{2x^2}\right\}\right] & ,\quad x<t\\ 0 & ,\quad x>t\end{cases}. \tag{19}$$

The contours of the solution (19) in the (x,z) plane are illustrated in Figures 1 and 2 for different values of time t, where $\overline{C}(x,z,t)=C(x,z,t)/Q$. The solution has a discontinuity at $x=t$ for small values of t. Noting the transformation (18), it is obvious that the line $x-t=$ constant corresponds to $x^*-Ut^*=$constant, so, due to this characteristic, $dx^*/dt^*=U$, i.e. the concentration of particles travels with a speed U away from the source. This is illustrated for two sample values of $h\,(=0,5)$ in Figures 1 and 2, respectively. The source of particles in this case does not depend on the time. For $x<t$, the concentration decreases away from the source and spreads further from the source as t increases. For $x>t$, there are no particles. This situation applies for all values of the height of the source above ground level. For small values of the time t, the proximity of the front at $x=t$ to the source causes the particles to diffuse upwards, (Figures 1,2(a), (b), (c)). For large values of the time t, the discontinuity at $x=t$ has no effect on the distribution of the particles. When the source is situated on the ground, the concentration on the ground is strong but as the height increases, the particles spread over a larger area both horizontally and vertically. When $t\to\infty$, the distribution of particles approaches the steady state solution (Figures 1(f) and 2(f)).

$$(ii)\quad f(t)=1-e^{-\lambda t}\ ,\quad \lambda>0$$

The solution (17) becomes

$$C(x,z,t)=\begin{cases}\dfrac{Q}{2\sqrt{2\pi}\,x}\left(1-e^{-\lambda(t-x)}\right)\left[\exp\left\{\dfrac{-(h+z)^2}{2x^2}\right\}+\exp\left\{\dfrac{-(h-z)^2}{2x^2}\right\}\right], & x<t\\ 0, & x>t\end{cases}. \tag{20}$$

The contours of the solution (20) in the (x,z) plane are illustrated for different values of the time t and decay factor λ in Figures 3– 6, where $\overline{C}(x,z,t)=C(x,z,t)/Q$. We note that the strength of the source in this case depends on the coefficient of decay λ as well as on the time. Figures 3 and 4 illustrate the profiles of the solution for a fixed value of $\lambda=10$ and different values of the time t with two different heights of the source. It is clear that there is no discontinuity in the distribution of the particles in the (x,z) plane whatever the values of the time and height.

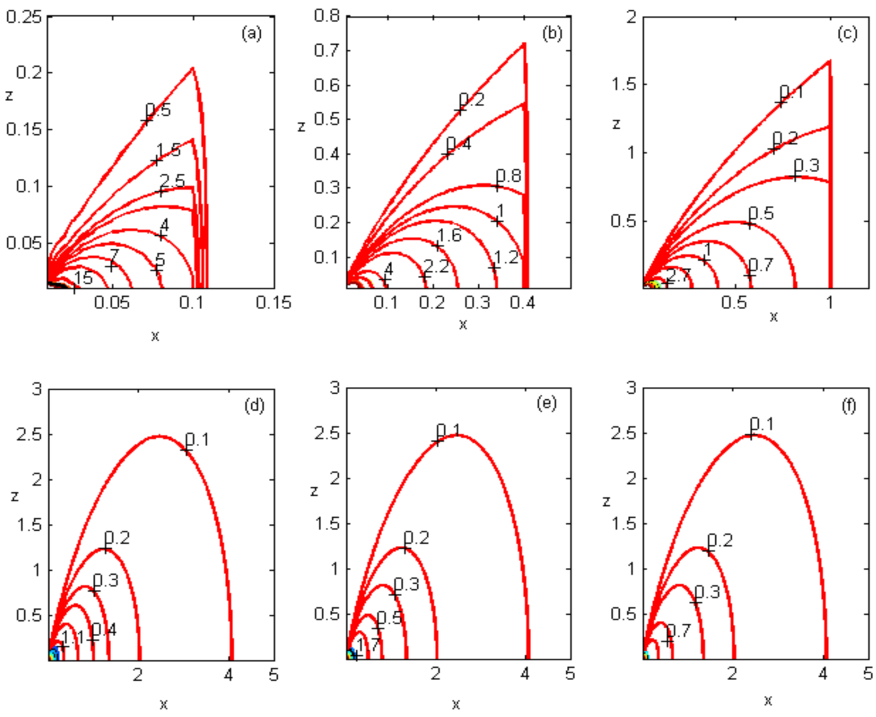

Figure 1. The isolines of the concentration $\overline{C}(x,z,t)=C(x,z,t)/Q$ in the (x,z) plane when $f(t)=H(t)$ and $h=0$, for different values of the time: (a) $t=0.1$, (b) $t=0.4$, (c) $t=1$, (d) $t=5$, (e) $t=7.5$, and (f) $t=10$. Note the precipitation of the particles on the ground as the time increases. Note the position of the characteristic $x=t$ as t is increased.

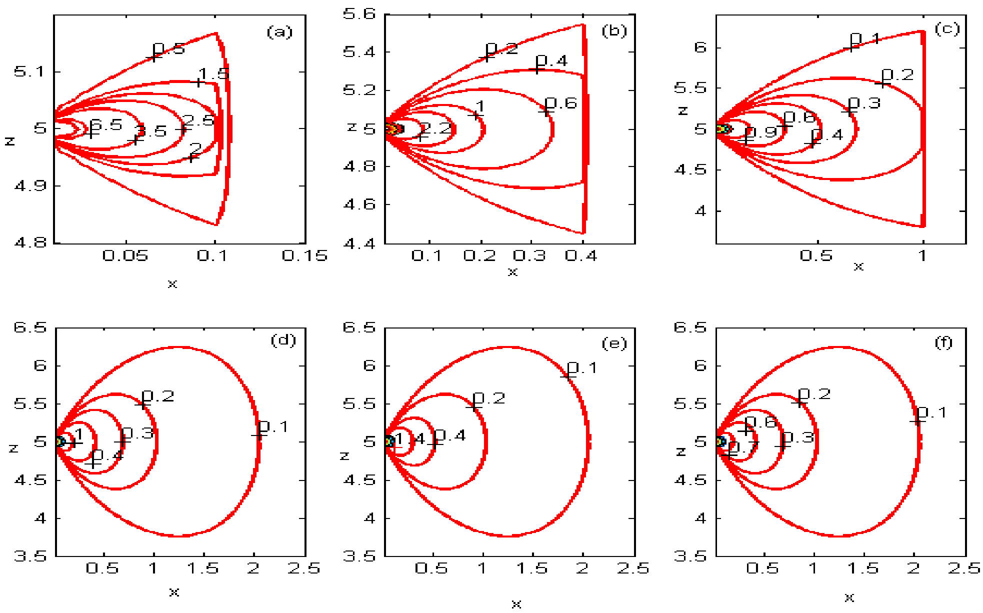

Figure 2. The isolines of the concentration $\overline{C}(x,z,t)=C(x,z,t)/Q$ in the (x,z) plane when $f(t)=H(t)$ and $h=5$, for different values of the time: (a) $t=0.1$, (b) $t=0.4$, (c) $t=1$, (d) $t=5$, (e) $t=7.5$, and (f) $t=10$. Compare Figures 1 and 2 to notice the influence of increasing the height of an industrial chimney.

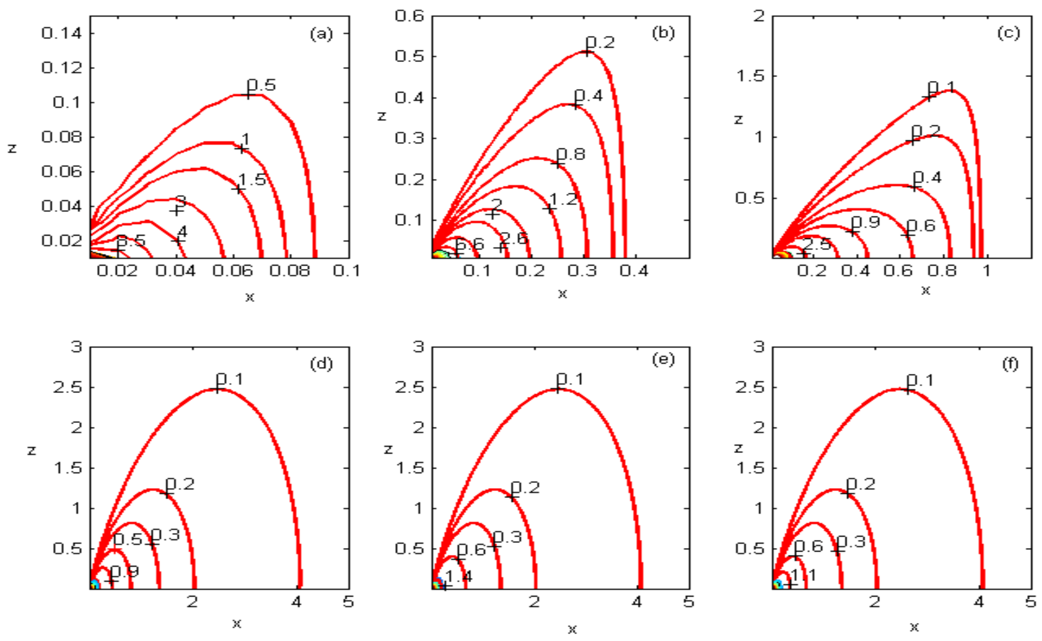

Figure 3. The isolines of the concentration $\overline{C}(x,z,t)$ in the (x,z) plane when $f(t)=1-e^{-\lambda t}$ for $h=0$, and $\lambda=10$ and the time t takes the values $0.1, 0.4, 1, 5, 7.5,$ and 10 in (a), (b), (c), (d), (e) and (f), respectively.

For large values of the time t, the distribution of the particles converges to the steady state solution (Figures 3 and 4(f)). Figures 5 and 6 show the profiles of the concentration in the space for a fixed value of the time $t=1$ and different values of the decay factor λ for two different values of the height of the source. For small values of λ, the strength of the source is weak and its ability to push the particles far away from the origin is not strong. When $\lambda\to\infty$, the spread of the particles approaches the distribution of particles in case (i). Comparison between the two cases of the function $f(t)$ shows that the dependence of the source on the time has an influence on the distribution of the particles in space. This applies to all values of the height (Figures 1 – 6).

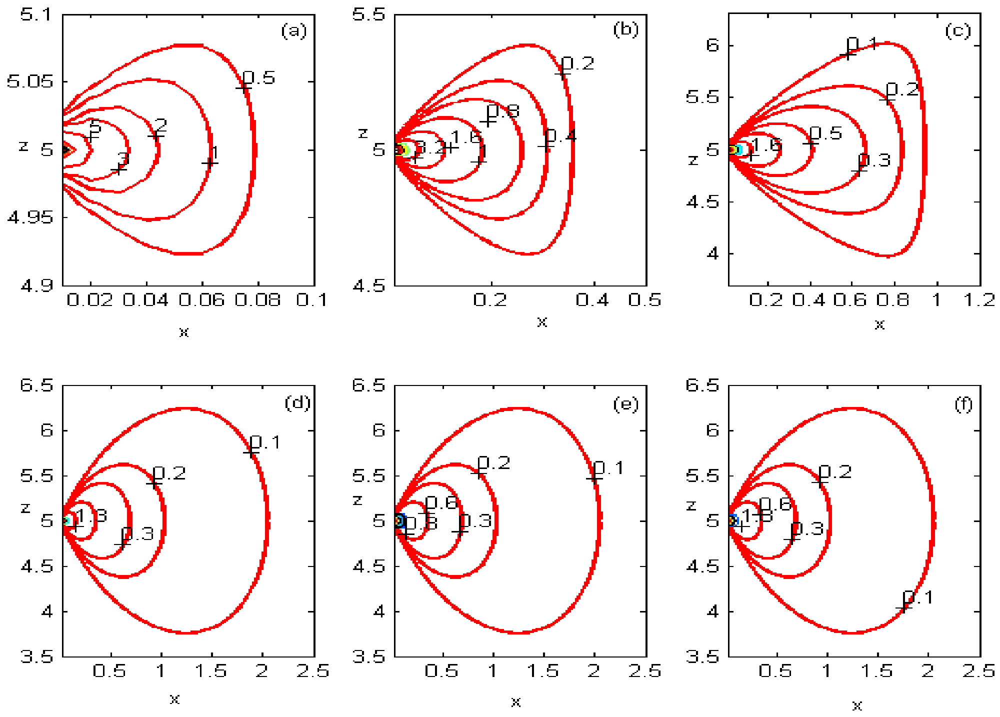

Figure 4. The isolines of the concentration $\overline{C}(x,z,t)$ in the (x,z) plane when $f(t)=1-e^{-\lambda t}$ for $h=5$, and $\lambda=10$ and the time t takes the values 0.1, 0.4, 1, 5, 7.5, and 10 in (a), (b), (c), (d), (e) and (f), respectively.

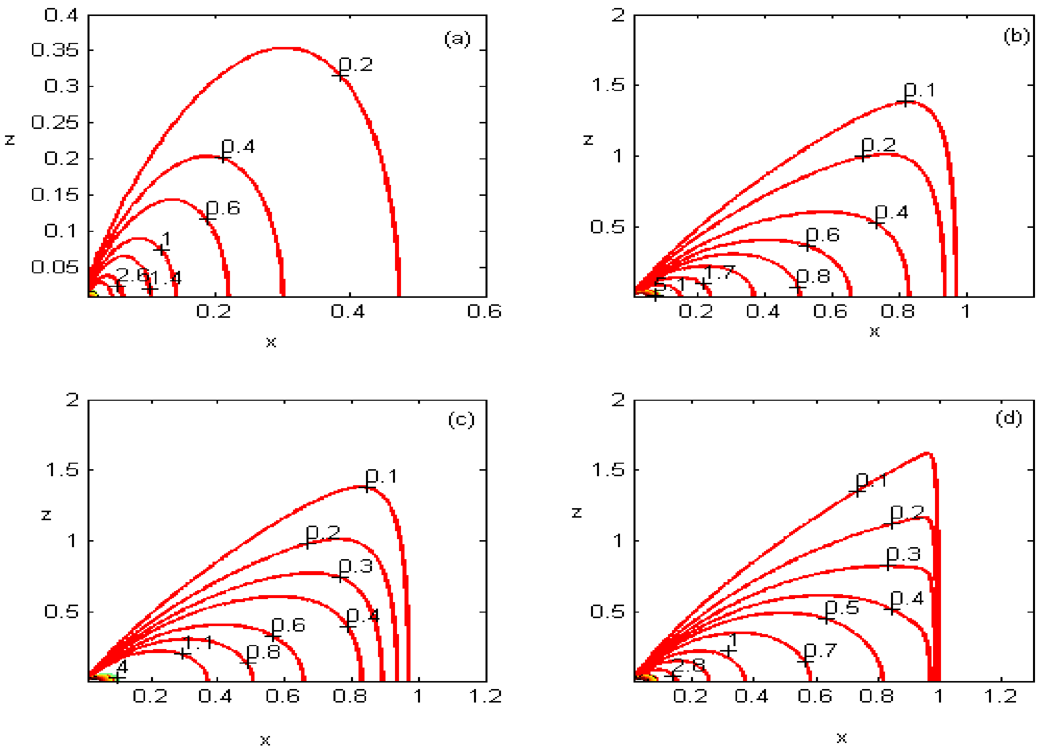

Figure 5. The isolines of the concentration $\overline{C}(x,z,t)$ in the (x,z) plane when $f(t)=1-e^{-\lambda t}$ for $h=0$, and $t=1$ and the factor λ takes the values 0.5, 1, 10, and 100 in (a), (b), (c) and (d), respectively. Note the increasing of the coefficient of decay.

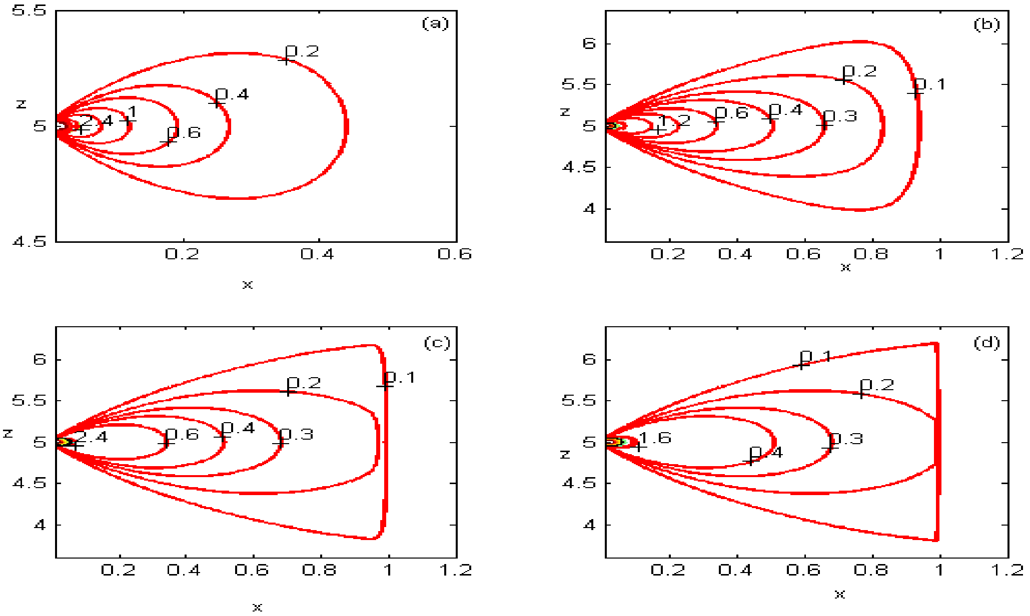

Figure 6. The isolines of the concentration $\overline{C}(x,z,t)$ in the (x,z) plane when $f(t)=1-e^{-\lambda t}$ for $h=5$, and $t=1$ and the factor λ takes the values 1, 10, 100, and 1000 in (a), (b), (c) and (d), respectively. Note when $\lambda\to\infty$, case 1(i) deduced from this case.

3.2. Case (2): Solution in the case of longitudinal diffusion ($\beta=\gamma=0,\ \alpha\neq 0$)

In this case, the longitudinal diffusion $(D_{x^*x^*})$ is present, and both vertical and latitudinal diffusions $(D_{z^*z^*}$ and $D_{y^*y^*})$, respectively, are absent. The solution in this case is given by

$$C(x,t)=\frac{Q}{\Gamma\left(\frac{1}{\alpha}\right)}\int_0^t g(t-s)\,s^{\left(\frac{1}{\alpha}-1\right)}\exp\left(\frac{-x}{\alpha s}\right)ds\ , \tag{21}$$

where x and Q are defined in (16), t is given in (18), $\Gamma(s)$ is the Gamma function, and $g(t)$ is the inverse Laplace transform of $F(\omega)$ with respect to the time t, given by

$$g(t)=L^{-1}\left\{\omega^{\frac{1}{\alpha}}F(\omega)\right\}.$$

$F(\omega)$ is the Laplace transform of the function $f(t)$ with respect to t defined by

$$F(\omega)=\int_0^\infty f(t)e^{-\omega t}dt.$$

The behavior of the concentration in this case depends on the source $f(t)$ as represented by $g(t)$ in (21). This expression gives the solution of this case in closed form. The model does not depend on height, z, or distance y. It then represents a source fixed along the z-axis at origin and having an infinite length ($z\geq 0$). The nature of the solution (21) is studied by using two examples of the function $f(t)$.

$$(i)\ f(t)=H(t)=\begin{cases}1 & ,t>0\\ 0 & ,\ t<0\end{cases}.$$

In this case, the solution (21) reduces to

$$C(x,t)\ =\ \frac{Q}{\Gamma\left(\frac{1}{\alpha}\right)}\left\{\Gamma\left(\frac{1}{\alpha}\right)-\gamma\left(\frac{1}{\alpha},\frac{x}{\alpha t}\right)\right\}\quad ;\ \alpha>1, \tag{22}$$

where $\gamma(a,x)$ is the Incomplete Gamma function.

For the special cases $(x=0\ ,\ t=0\ ,\ x\to\infty,\ t\to\infty)$, the solution (22) gives

$$C(0,t) = \frac{Q}{\Gamma(2\mu)}\{\ \Gamma(2\mu) - \gamma(2\mu,0)\ \} = Q,$$

$$C(x,0) = \frac{Q}{\Gamma(2\mu)}\{\ \Gamma(2\mu) - \gamma(2\mu,\infty)\ \} = 0,$$

$$C(x,t) \to \frac{Q}{\Gamma(2\mu)}\{\ \Gamma(2\mu) - \gamma(2\mu,\infty)\ \} \to 0 \qquad ;\ \text{as}\ x\to\infty,$$

$$C(x,t) \to \frac{Q}{\Gamma(2\mu)}\{\ \Gamma(2\mu) - \gamma(2\mu,0)\ \} \to Q \qquad ;\ \text{as}\ t\to\infty.$$

These asymptotic values confirm the initial and boundary conditions for $C(x,t)$ of this case. Another special case occurs when $\alpha=2,$ when the solution (22) reduces to a simpler form. In such a case, the solution (22) becomes

$$C(x,t) = Q\ erfc\left(\sqrt{\frac{x}{2t}}\right) \qquad ;\ \alpha=2.0, \tag{23}$$

in which $erfc(y)$ is the complementary error function [19].

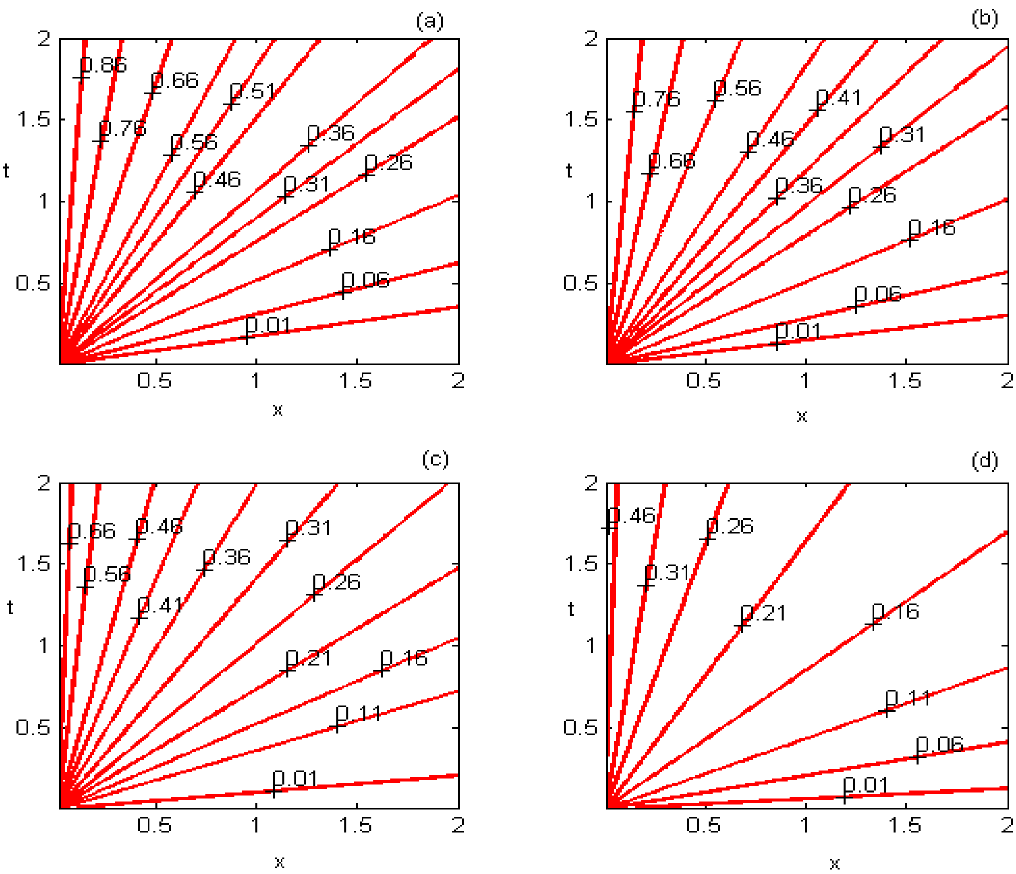

Figure 7. The isolines of the concentration $\overline{C}(x,t)$ of case 2(i) in the (x,t) plane for the function $f(t)=H(t)$ for some values of the diffusion parameter: $(a)\ \alpha=1.5$, $(b)\ \alpha=2,$ $(c)\ \alpha=4$, and $(d)\ \alpha=10$. Note the increasing of the longitudinal diffusion parameter.

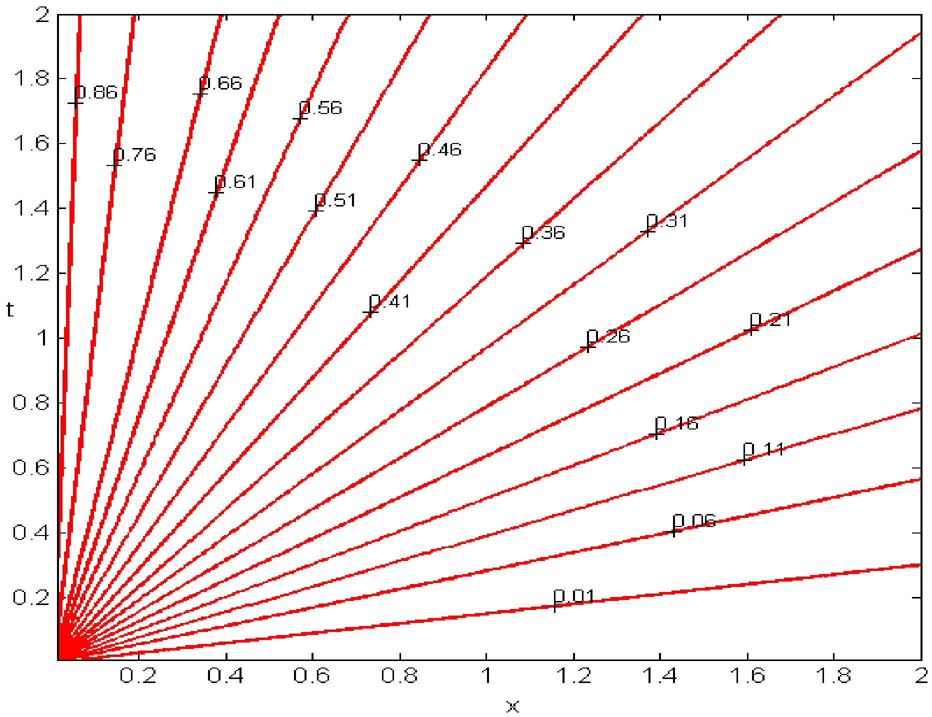

Figure 8. The isolines of the concentration $\overline{C}(x,t)$ in the (x,t) plane for the function $f(t) = H(t)$ for a special case of complementary error function when $\alpha = 2$, in the case when longitudinal diffusion only is present. Compare this with Figure 7 (b).

The contours of the solutions (22) and (23) in the (x,t) plane are illustrated for different values of the diffusion component α in Figure 7 and for $\alpha = 2$ in Figure 8. It is clear from Figure 7 that at every point of the space in the (x,t) plane, the presence of particles decreases whenever the diffusion parameter α increases. For a fixed value of the distance x, the concentration increases with time t. Moreover, at a specific time t, the concentration of particles decreases when we move further away from the source. This last situation can happen in real life because as we move further away from the source, there will not have been enough force to carry a mass/ large numbers of particles that far. The values of α chosen for Figure 8 are made for ease of comparison between the solutions (22) and (23). If we compare the two figures, we see that the computations for both cases are consistent.

$$(ii) \quad f(t) = 1 - e^{-\lambda t} \quad , \quad \lambda > 0$$

The solution for this source is given by

$$C(x,t) = \frac{Q\lambda}{\Gamma(2\mu)\Gamma(2-2\mu)} \int_0^t \left(\frac{t}{s} - 1\right)^{1-2\mu} e^{\frac{-2\mu x}{s}} M(1, 2-2\mu, \lambda(s-t))ds, \tag{24}$$

where $M(a,b,x)$ is a confluent hypergeometric function [19].

The contours of the solution (24) in the (x,t) plane are presented in Figures 9 and 10 for different values of the decay parameter λ and the two specific values of the coefficients of the longitudinal diffusion; $\alpha = 2$ and $\alpha = 10$, respectively. The quantity of particles at every point of the (x,t) plane increases whenever the decay factor λ increases for a fixed value of longitudinal diffusion α. This situation must happen in real life because when the coefficient of the decay increases, the strength of the source increases. When the values of α and λ are fixed, then the concentration increases as the time increases for a fixed distance x. Furthermore, at a specific time t, for fixed strength of the source and longitudinal diffusion, the concentration decreases as we move further away from the source.

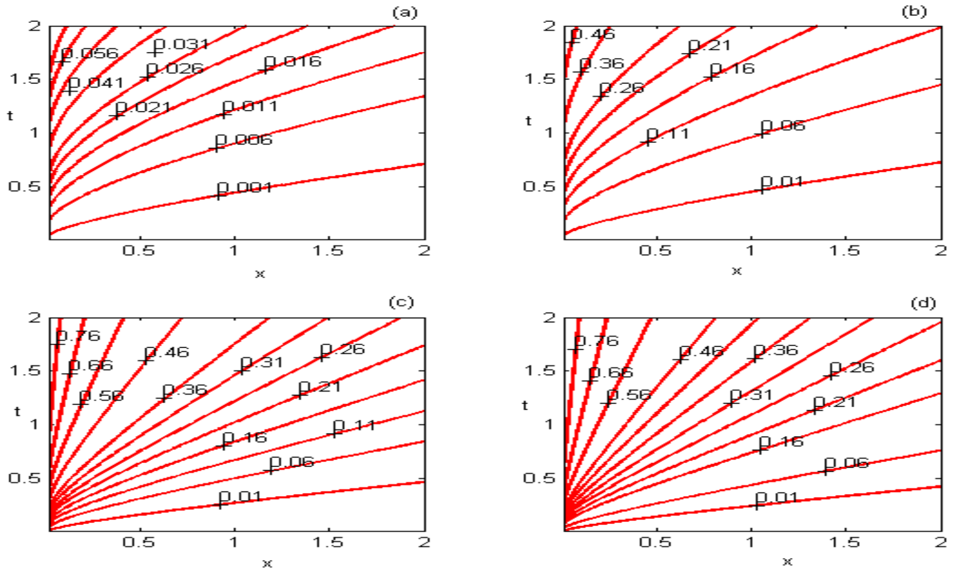

Figure 9. The profile of the concentration $\overline{C}(x,t)$ for a fixed value of $\mu = 0.25$ ($\alpha = 2$) and different values of the decay factor: $(a)\,\lambda = 0.1$, $(b)\,\lambda = 1$, $(c)\,\lambda = 6$, and $(d)\,\lambda = 10$.

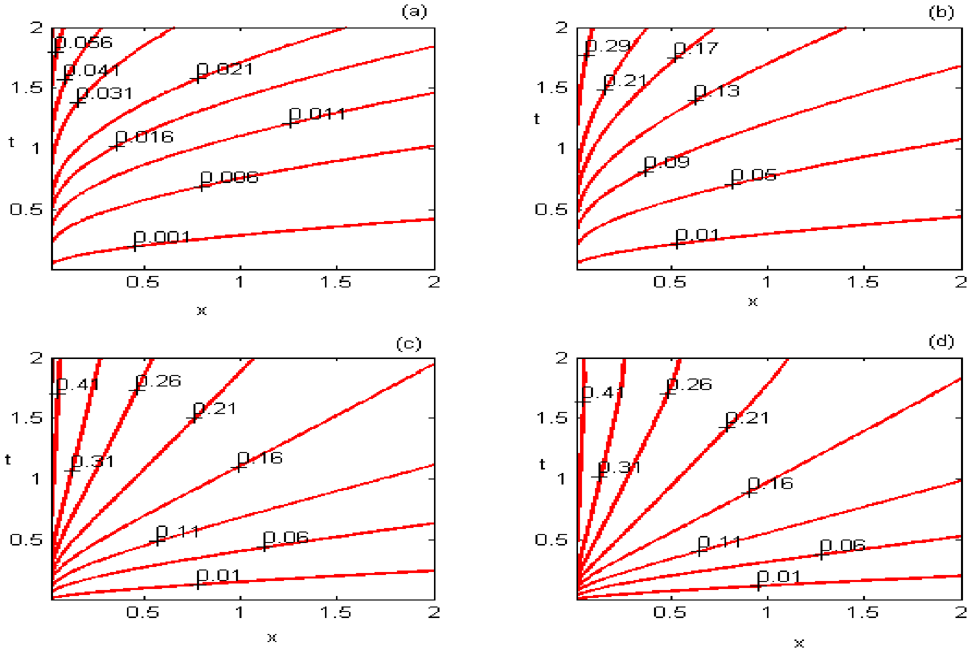

Figure 10. The profile of the concentration $\overline{C}(x,t)$ for a fixed value of $\mu = 0.05$ ($\alpha = 10$) and different values of decay factor: $(a)\,\lambda = 0.1$, $(b)\,\lambda = 1$, $(c)\,\lambda = 5$, and $(d)\,\lambda = 10$.

As shown in Figures 9 and 10, for small values of λ and fixed α, the diffusion of the particles in the horizontal direction is very weak and the particles need more time to diffuse in the $x-$direction, (Figures 9 (a, b), and 10 (a, b)). As λ increases, the diffusion steadily increases. For large values of the decay factor λ, the concentration approaches the steady state solution. Large values of λ result in a strong source and whenever these values increase further the strength of the source converges to the steady state in case (i). For a fixed value of the decay factor, at every point in the plane the concentration when $\alpha = 2$ is more than that when $\alpha = 10.$ Comparison between the two cases of $f(t)$ in the presence of longitudinal diffusion shows that the force of the source in case (i) is stronger than that for the source in case (ii). But the two will be identical for large values of the decay factor λ, for $t > 0$, when the strengths of the two sources become equal. The spread of particles in the horizontal direction in case (ii) is weaker than that in case (i) for $t > 0.$ For large values of the time, the unsteady state solution approaches the steady state results obtained by Sharan *et al* [1].

4. Conclusion

The diffusion of dust particles emitted from a fixed source in the atmosphere in the absence of the settling velocity has been studied mathematically. The study found the solution of the time-dependent diffusion equation in the presence of a point source whose strength is dependent on time. The solution reduces to simpler forms in special cases where the solution can be obtained in explicit expression. In case (1), diffusion in both the longitudinal and latitudinal directions was neglected. The dependence of the distribution of dust on the time variation of the source is investigated for two different functions. When the source is strong for small times, the solution shows a discontinuity. In case (2), the diffusion in the y and z directions was ignored. The profiles of the solutions showed that the concentration of dust particles in the (x,t) plane depends on the parameter of diffusion in the direction of the wind. The strong presence of this parameter led to the distribution of the dust in a larger area. The solution approaches the steady state solution of the system.

References

1. Sharan, M., Singh, M. and Yadav, A. Mathematical model for atmospheric dispersion in low winds with eddy diffusivities as linear functions of downwind distance. *Atmospheric Environment*, 1996, **30**, 1137-1145.
2. Taylor, G. Diffusion by continuous movements. *Proceedings of the London Mathematical Society* A, 1921, **20**, 196 - 212.
3. Abbott, J. and Francis, J. Saltation and suspension trajectories of solid grains in a water stream. *Philosophical Transactions of the Royal Society of London*, 1977, **A284**, 225-253.
4. Eltayeb, I. and Hassan, M. On the non-linear evolution of sand dunes. *Geophysical Journal of the Royal Astronomical Society,* 1981, **65**, 31-45.
5. Koch, W. A solution of the two-dimensional atmospheric diffusion equation with height-dependent diffusion coefficient and height-dependent wind speed and eddy diffusivities. *Atmospheric Environment*, 1989, **23**, 1729-1732.
6. Hill, J. Similarity solutions for non-linear diffusion a new integration procedure. *Journal of Engineering Mathematics*, 1989, **23**,141-155.
7. Eltayeb, I. and Hassan, M. Two-dimensional transport of dust from an infinite line source at ground level. *Geophysical Journal International*, *1992*, **110**, 571-576.
8. Lin, J. and Hildemann, L. Analytical solutions of the atmospheric diffusion equation with multiple sources and height-dependent wind speed and eddy diffusivities. *Atmospheric Environment,* 1996, **30**, 239-254.
9. Lin, J. and Hildemann, L. A generalized mathematical scheme to analytically solve the atmospheric diffusion equation with dry deposition. *Atmospheric Environment*, 1997, **31**, 59-71.
10. AL-Mashrafi, K. Diffusion of dust particles emitted from a fixed source. *International Journal of Applied Mathematical Research*, 2015, **4(4)**, 454-465.
11. AL-Mashrafi, K. The influence of longitudinal diffusion on the transport of dust particles emitted from a fixed source. *International Journal of Applied Mathematical Research*, 2016, **5(1)**, 11-17.
12. Roberts, O. 1923. The theoretical scattering of smoke in a turbulent atmosphere. *Proceedings of the Royal Society of London.*,1923, **A 104**, 640-654.
13. Smith, F. The diffusion of smoke from a continuous elevated point source into a turbulent atmosphere. *Journal of Fluid Mechanics*, 1957, **2**, 49-76.
14. Gillette, D. Threshold friction velocities for dust production for agricultural soils. *Journal of Geophysical Research,* 1988, **93**, 12645-12662.
15. Eltayeb, I. and Hassan, M. Diffusion of dust particles from a point source above ground level and a line source at ground level. *Geophysical Journal International*, 2000, **142**, 426-438.
16. Hassan, M. and Eltayeb, I. Time-dependent transport of dust. *Journal of Geophysical Research*, 1991, **96**, 9337-9339.
17. Gillette, D. and Passi, R. Modelling dust emission caused by wind erosion. *Journal of Geophysical Res*earch, 1988, **93**, 14233-14242.
18. Pasquill, F. Atmospheric Diffusion, Van Nostrand, Princeton. 1962
19. Abramowitz, M. and Stegun, I. *1965 Handbook of Mathematical Functions, with Formulas, Graphs and Mathematical Tables*, New York, Dover.

Received 26 February 2017
Accepted 27 July 2017