Behavior of Small Variable Mass Particle in Electromagnetic Copenhagen Problem

Abdullah A. Ansari

International Center for Advanced Interdisciplinary Research (ICAIR), Sangam Vihar, New Delhi, India. Email: icairndin@gmail.com.

ABSTRACT: The paper presents the behavior of the motion properties of the variable mass test particle (third body), moving under the influence of the two equal primaries having electromagnetic dipoles. These primaries move on the same circular path around their common center of mass in the same plane. We have determined the equations of motion of the test particle whose mass varies according to Jean's law. Using the system of equations of motion we have evaluated the locations of equilibrium points, their movements and basins of the attracting domain. Finally, we examine the stability of these equilibrium points, all of which are unstable.

Keywords: Variable mass; Electromagnetic dipoles; Copenhagen problem; Attracting domain.

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سلوك جسيم ذو كتلة خاضعة لتغيرات صغيرة في مشكلة كوبنهاجن الكهرومغناطيسية

عبدالله عبدالجبار أنصاري

الملخص: تقدم هذه الورقة العلمية سلوك خصائص حركة جسيم ذو كتلة متغيرة (الجسم الثالث) تحت تأثير جسمان اوليان متساويان لهما قطبان مغنتسيان. الجسمان الأوليان يتحركان على نفس المصار الدائري مركزه مركزهما الكتلي المشترك المتواجد في نفس المصتح. قمنا بإتقييم معادلات حركة الجسيم حيث نفرض ان كتلته تتغير حسب قانون جان . باستعمال نضام معادلات الحركة، قمنا بحساب أماكن نقاط التوازن ، حركتهما و أحواض مجالات الجذابة. في آخر العمل قمنا بالتترق لمسألة إستقرار نقاط التوازن و وجدنا أن كلها غير مستقرة.

الكلمات المفتاحية: كتلة متغيرة، ثنائي الأقطاب كهرومغناطيسي، مسألة كوبنهاق، مجال ذو جاذبية.

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1. Introduction

Celestial Mechanics is a part of applied mathematics; basically it is the study of the motion properties of satellites under the influence of celestial bodies. The motion of these satellites or spacecraft is affected by the celestial bodies in many ways, such as by the different shapes of the bodies (i.e. heterogeneous body, homogeneous body, irregular triaxial bodies, oblate body, finite straight segments, cylindrical shape, etc.), radiation pressure, resonances, magnetic dipoles, charged bodies, Yarkovskii effects, the albedo effect, variable mass effects, etc. For more details, see [1] and [2].

An artificial body (satellite or spacecraft) is also perturbed by the electromagnetic dipoles which are studied by [3-5]. [3] presented the motion properties of charged particles as equilibrium points and their stability in the field of two magnetic dipoles. [4] has investigated the three-dimensional equilibrium points in a magnetic-binary system and then, by using the method of characteristic exponent, examined their stability. In continuation of the work of [4], [5] studied the basins of attraction in the electromagnetic Copenhagen problem.

The Copenhagen problem where two primary bodies of equal mass are moving on the same circular path in the same plane is also commonly studied in celestial mechanics. [6] used the methods of chaotic scattering to investigate this problem. [7] used studied the asymptotic orbits and terminations of families in the Copenhagen restricted three-body problem. [8] studied the Copenhagen problem with quasi-homogeneous potential or Manev-type potential and illustrated the equilibrium locations and the regions of permitted motion. [9] derived the translational-rotational equations of motion and investigated the stability of its equilibrium points. [10] have investigated equilibrium positions as well as the basins of attraction in the Copenhagen restricted three-body problem numerically.

After careful study of the literature presented, we became interested in studying and extending this topic with variable mass. Variable mass is an interesting topic in celestial mechanics and dynamical astronomy, which is widely studied in the restricted problem (two-body, three-body, four-body and five-body) [11-21].

However, no article yet exists in the literature addressing the combination of electromagnetic dipole and variable mass.

This paper is organized as follows. The literature review is presented in section 1. The equations of motion are presented in section 2 while section 3 contains the investigation of equilibrium points and basins of attracting domain. The stability of these equilibrium points are examined in section 4. Finally the manuscript completes with a conclusion in section 5.
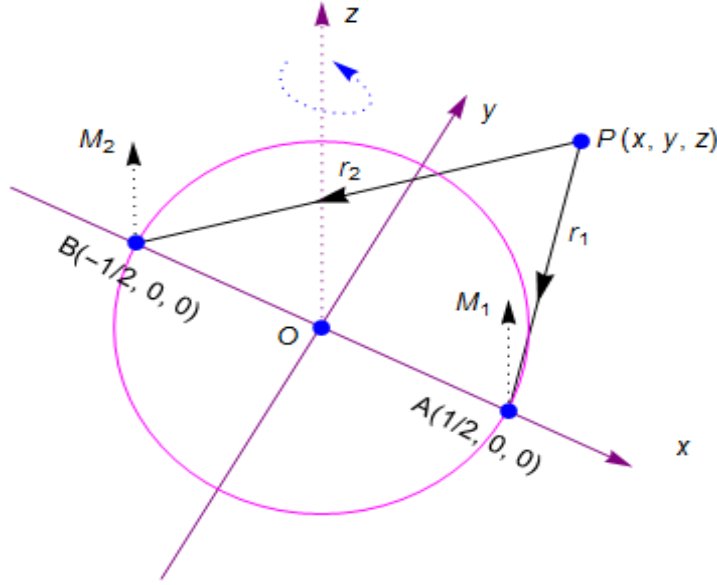

Figure 1. Geometric configuration of the electromagnetic Copenhagen problem

2. Equations of motion

The well-known Copenhagen problem where two bodies of equal masses m_1 and m_2 both are taken as point masses which are moving in circular orbits on the same circular path around their common center of mass. In this paper two bodies are also taken as electromagnetic dipoles with magnetic moments M_i (i = 1, 2). Now we are interested to determine the equations of motion of the third variable mass (m) body of charge q (Figure 1) with dimensionless variables in the synodic coordinate system where the variation of mass of the test particle originates from one point and has zero momenta ([17] and [3]) which reveals as:

$$\frac{\dot{m}}{m}(\dot{x} - \omega\, y) + (\ddot{x} - f_1\dot{y} + g_1\dot{z}) = \omega^2 x + \frac{q\,\omega}{m\,c}\left(x\frac{\partial A_2}{\partial x} - y\frac{\partial A_1}{\partial x} + A_2\right),$$

$$\frac{\dot{m}}{m}(\dot{y} + \omega\, x) + (\ddot{y} - h_1\dot{z} + f_1\dot{x}) = \omega^2 y + \frac{q\,\omega}{m\,c}\left(x\frac{\partial A_2}{\partial y} - y\frac{\partial A_1}{\partial y} - A_1\right),$$

$$\frac{\dot{m}}{m}\dot{z} + (\ddot{z} - g_1\dot{x} + h_1\dot{y}) = \frac{q\,\omega}{m\,c}\left(x\frac{\partial A_2}{\partial z} - y\frac{\partial A_1}{\partial z}\right). \tag{1}$$

where

$$f_1 = \left(2\,\omega + \hat{k}\ Curl\ A\right)\frac{q}{m\,c}, \quad g_1 = \left(\hat{j}\ Curl\ A\right)\frac{q}{m\,c}, \quad h_1 = \left(\hat{i}\ Curl\ A\right)\frac{q}{m\,c}, \tag{2}$$

with

$$A = (A_1, A_2, A_3) = \left(\frac{\bar{M}_1 \times \bar{r}_1}{r_1^3} + \frac{\bar{M}_2 \times \bar{r}_2}{r_2^3} \right),$$

$$\bar{r}_1 = (x - 0.5, y, z), \bar{r}_2 = (x + 0.5, y, z),$$

$$\bar{M}_1 = (0, 0, 1), \qquad \bar{M}_2 = (0, 0, \lambda),$$

$$A_1 = \left(-\frac{y}{r_1^3} - \frac{\lambda\,y}{r_2^3} \right), \; A_2 = \left(\frac{x - 0.5}{r_1^3} + \frac{\lambda(x + 0.5)}{r_2^3} \right), \qquad A_3 = 0,$$

and also ω is the mean motion of the system, λ and c are constants.
We choose the unit of time and charge of dipoles such that $\omega = 1$ and $(q/mc) = 1$, hence Equations (1, 2) become as

$$\frac{\dot{m}}{m}(\dot{x} - y) + (\ddot{x} - f_2\dot{y} + g_2\dot{z}) = \Omega_x,$$

$$\frac{\dot{m}}{m}(\dot{y} + x) + (\ddot{y} - h_2\dot{z} + f_2\dot{x}) = \Omega_y,$$

$$\frac{\dot{m}}{m}\dot{z} + (\ddot{z} - g_2\dot{x} + h_2\dot{y}) = \Omega_z,$$

$$\tag{3}$$

with

$$\Omega = \frac{1}{2}(x^2 + y^2) + (xA_2 - yA_1) \tag{4}$$

$$f_2 = \left(2 + \hat{k}\ Curl\ A\right), g_2 = (\hat{j}\ Curl\ A), h_2 = (\hat{i}\ Curl\ A), \tag{5}$$

Due to variation of the mass of the infinitesimal body, we cannot study the properties of this body directly. Therefore we will use Jean's law,

$$\frac{dm}{dt} = -\lambda_1 m^{\lambda_2}, \tag{6}$$

where λ_1 is constant coefficient and λ_2 is within the limits $0.4 \le \lambda_2 \le 4.4$, for the stars of the main sequence. While for the rocket $\lambda_2 = 1$, therefore the mass of the rocket $m = m_0\,e^{-\lambda_1 t}$ varies exponentially where m_0 is the mass of the test particle at time $t = 0$. We will use the Meshcherskii space time transformations to preserve the dimension of the space and time

$$x = \lambda_3^{-1/2}\alpha, \; y = \lambda_3^{-1/2}\beta, \; z = \lambda_3^{-1/2}\gamma, \; dt = d\tau \tag{7}$$

where $\lambda_3 = m/m_0$, now we can express the components of velocity and acceleration as

$$\dot{x} = \lambda_3^{-1/2}\left(\dot{\alpha} + \tfrac{1}{2}\lambda_1\alpha\right), \; \dot{y} = \lambda_3^{-1/2}\left(\dot{\beta} + \tfrac{1}{2}\lambda_1\beta\right), \dot{z} = \lambda_3^{-1/2}\left(\dot{\gamma} + \tfrac{1}{2}\lambda_1\gamma\right), \tag{8}$$

$$\ddot{x} = \lambda_3^{-1/2}\left(\ddot{\alpha} + \lambda_1\dot{\alpha} + \tfrac{1}{4}\lambda_1^2\alpha\right), \; \ddot{y} = \lambda_3^{-1/2}\left(\ddot{\beta} + \lambda_1\dot{\beta} + \tfrac{1}{4}\lambda_1^2\beta\right), \ddot{z} = \lambda_3^{-1/2}\left(\ddot{\gamma} + \lambda_1\dot{\gamma} + \tfrac{1}{4}\lambda_1^2\gamma\right), \tag{9}$$

After using Equations (6, 7, 8 and 9) in Equation (3), we get

$$\left(\ddot{\alpha} - f_3\dot{\beta} + g_3\dot{\gamma} \right) = V_\alpha + V_1,$$

$$\left(\ddot{\beta} - h_3\dot{\gamma} + f_3\dot{\alpha} \right) = V_\beta + V_2,$$

$$\left(\ddot{\gamma} - g_3\dot{\alpha} + h_3\dot{\beta} \right) = V_\gamma + V_3, \tag{10}$$

where, $\quad V = \frac{1}{2}(\alpha^2 + \beta^2) + \frac{\lambda_1^2}{8}(\alpha^2 + \beta^2 + \gamma^2) + \lambda_3^{-1/2}(\alpha B_2 - \beta B_1)$ (11)

$$V_1 = \lambda_3^{3/2} \frac{\lambda_1}{2} \left(\beta \frac{\partial B_2}{\partial \alpha} - \beta \frac{\partial B_1}{\partial \beta} - \gamma \frac{\partial B_1}{\partial \gamma} \right),$$

$$V_2 = \lambda_3^{3/2} \frac{\lambda_1}{2} \left(-\alpha \frac{\partial B_2}{\partial \alpha} + \alpha \frac{\partial B_1}{\partial \beta} - \gamma \frac{\partial B_2}{\partial \gamma} \right),$$

$$V_3 = \lambda_3^{3/2} \frac{\lambda_1}{2} \left(\alpha \frac{\partial B_1}{\partial \gamma} + \beta \frac{\partial B_2}{\partial \gamma} \right),$$

$$f_3 = \lambda_3^{3/2} \left(2 \lambda_3^{-3/2} + \hat{k}.Curl\ B \right),\ g_3 = \lambda_3^{3/2} (\hat{j}.Curl\ B),\ h_3 = \lambda_3^{3/2}(\hat{\imath}.Curl\ B), \tag{12}$$

$$B = (B_1, B_2, B_3),$$

$$B_1 = \left(-\frac{\beta}{\ell_1^3} - \frac{\lambda\ \beta}{\ell_2^3} \right),\ B_2 = \left(\frac{\alpha - \lambda_3^{1/2}0.5}{\ell_1^3} + \frac{\lambda(\alpha + \lambda_3^{1/2}0.5)}{\ell_2^3} \right),\quad B_3 = 0,$$

$$\ell_1 = \sqrt{\left(\alpha - \lambda_3^{1/2}0.5 \right)^2 + \beta^2 + \gamma^2},\quad \ell_2 = \sqrt{\left(\alpha + \lambda_3^{1/2}0.5 \right)^2 + \beta^2 + \gamma^2} \tag{13}$$

3. Analysis of Equilibrium Points and Basins of Attracting Domain

3.1 Equilibrium points

We will put all the derivatives with respect to time on the left hand side of the system (10) to zero, hence

$$V_\alpha + V_1 = 0, \tag{14}$$

$$V_\beta + V_2 = 0, \tag{15}$$

$$V_\gamma + V_3 = 0. \tag{16}$$

After solving equations (14), (15) and (16), we can find the locations of equilibrium points. We have plotted the locations of equilibrium points numerically in α-β-plane as shown in Figures 2, 3 and 4.

(a) $\lambda_1 = 0$ and $\lambda_3 = 1$

(b) $\lambda_1 = 0.2$ and $\lambda_3 = 0.4$

(c) $\lambda_1 = 0.2$ and $\lambda_3 = 0.8$

(d) $\lambda_1 = 0.2$ and $\lambda_3 = 1.4$
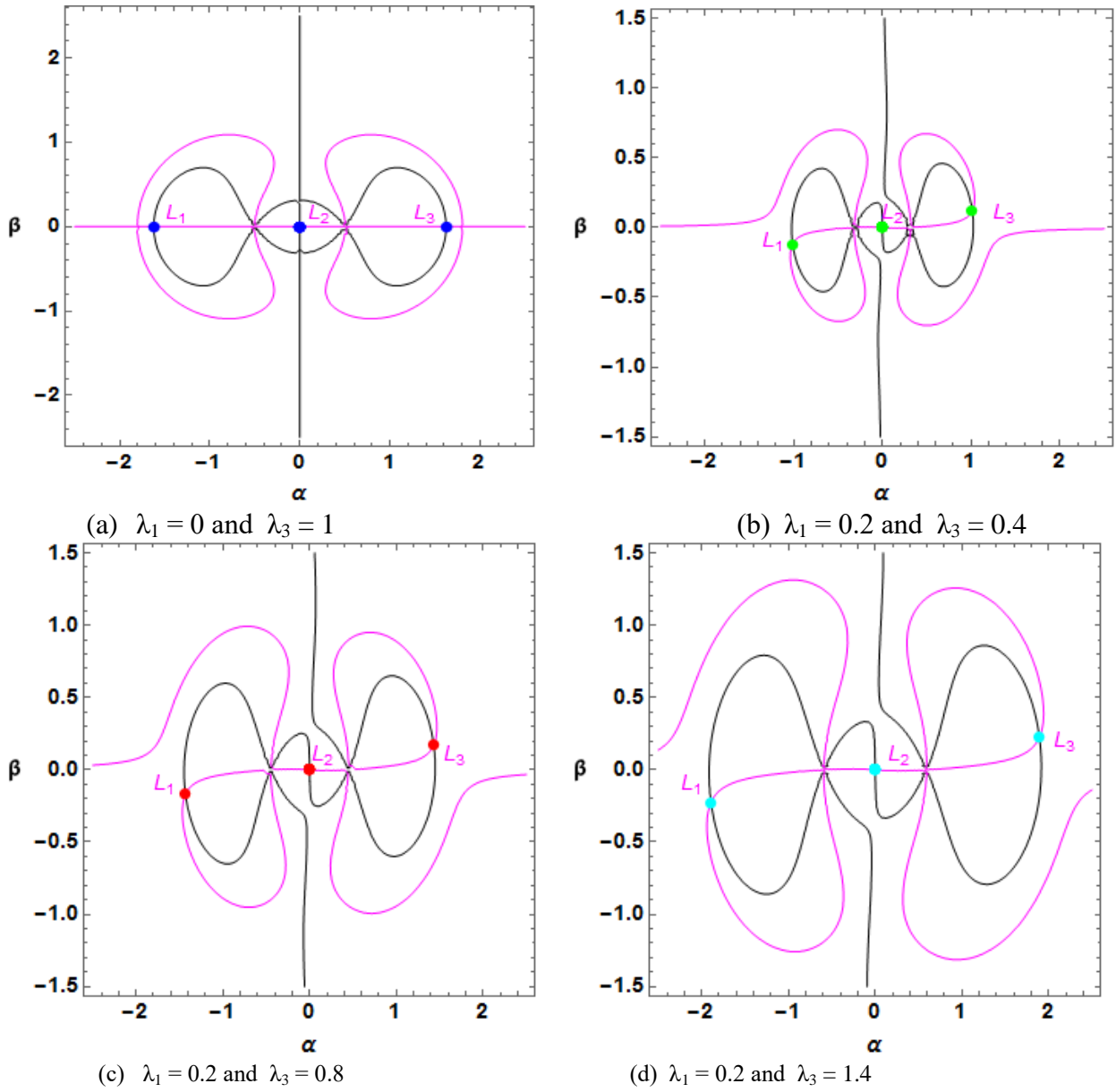
Figure 2. Locations of equilibrium points at $\lambda = 1$ in $\alpha\beta$-plane

(a) $\lambda_1 = 0$ and $\lambda_3 = 1$

(b) $\lambda_1 = 0.2$ and $\lambda_3 = 0.4$

(c) $\lambda_1 = 0.2$ and $\lambda_3 = 0.8$

(d) $\lambda_1 = 0.2$ and $\lambda_3 = 1.4$
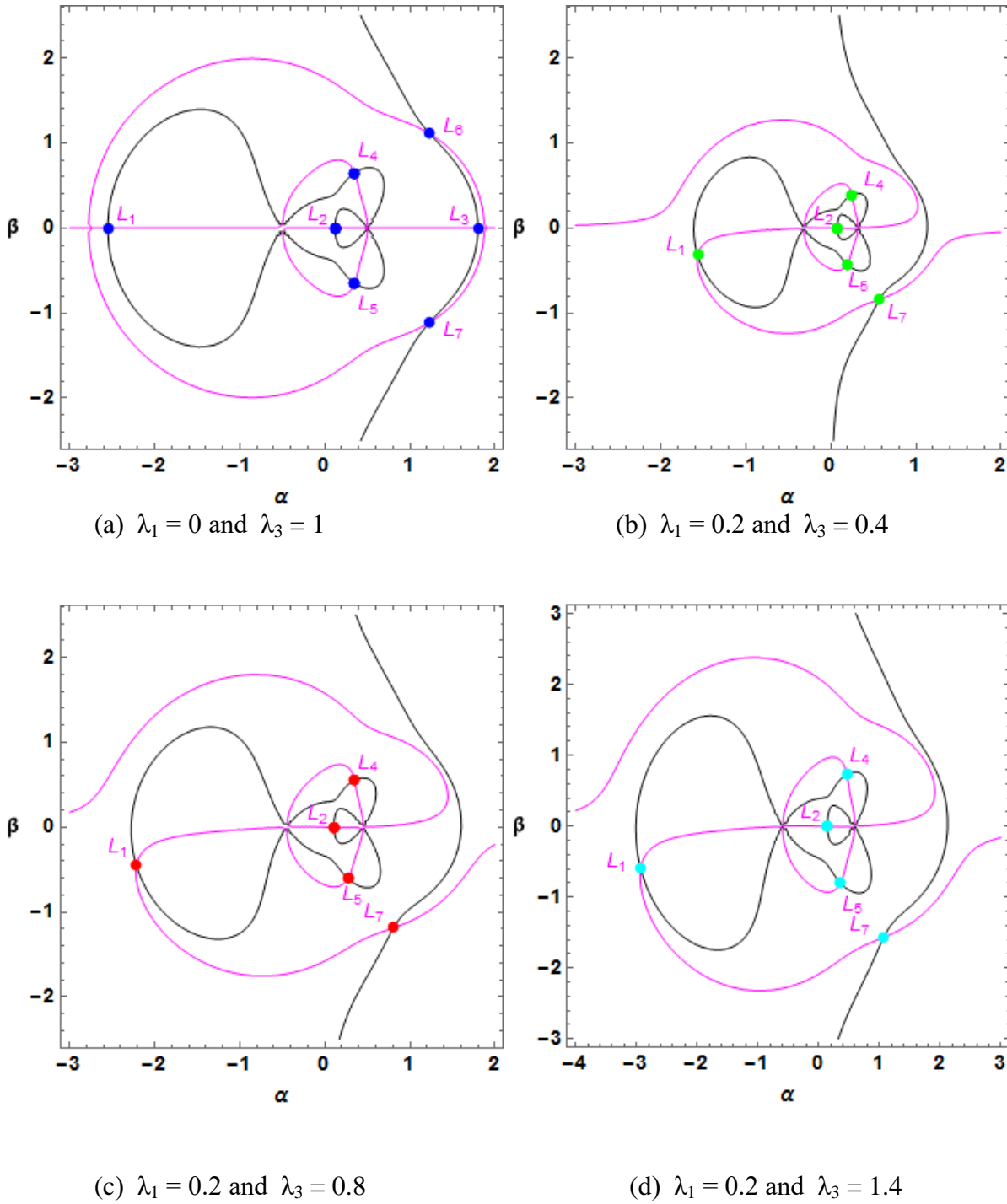
Figure 3. Locations of equilibrium points at $\lambda = 7$ in α β-plane.

(a) $\lambda_1 = 0$ and $\lambda_3 = 1$

(b) $\lambda_1 = 0.2$ and $\lambda_3 = 0.4$

(c) $\lambda_1 = 0.2$ and $\lambda_3 = 0.8$

(d) $\lambda_1 = 0.2$ and $\lambda_3 = 1.4$
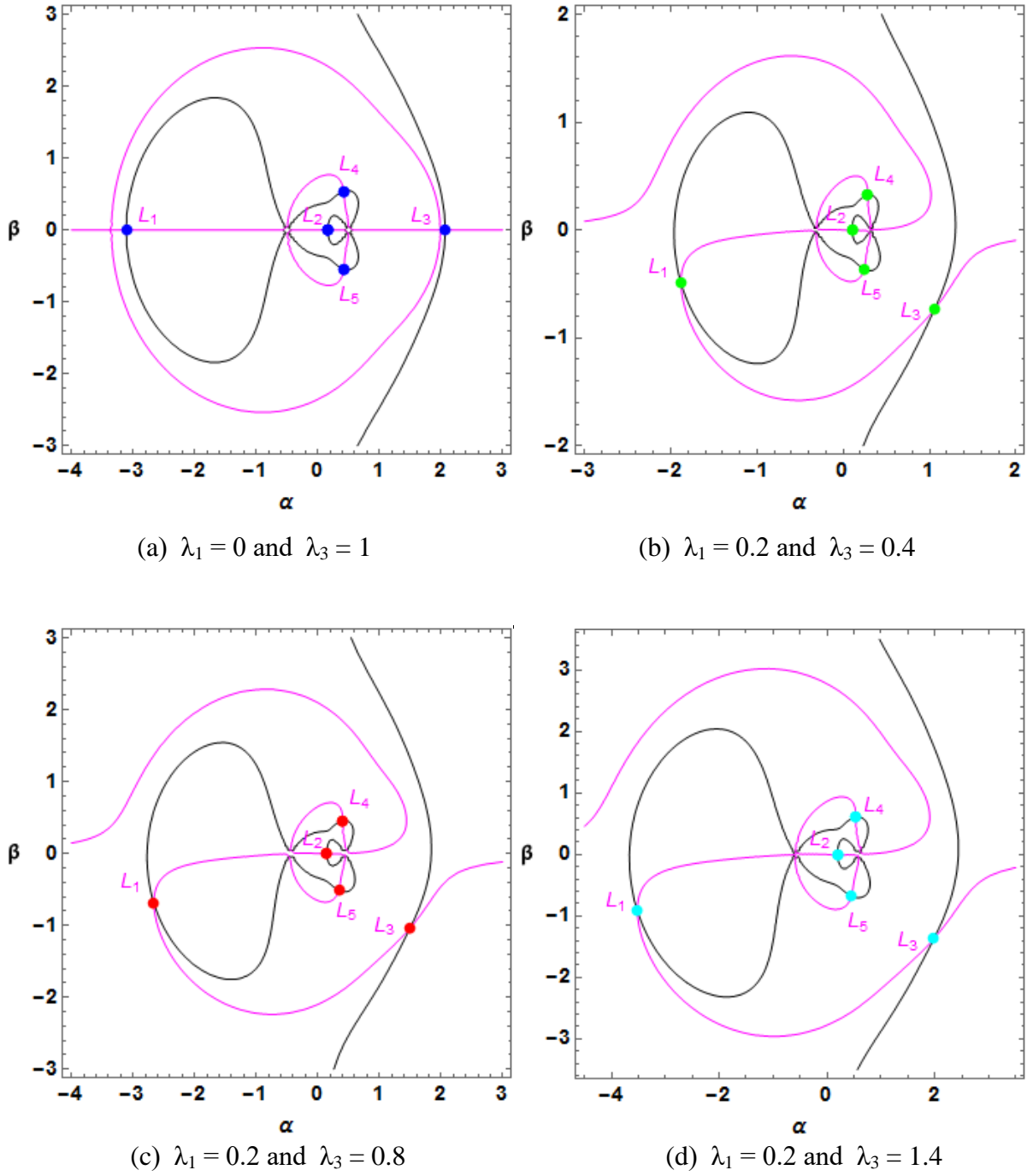
Figure 4. Locations of equilibrium points at $\lambda = 15$ in $\alpha \beta$-plane.

(a) $\lambda = 1$

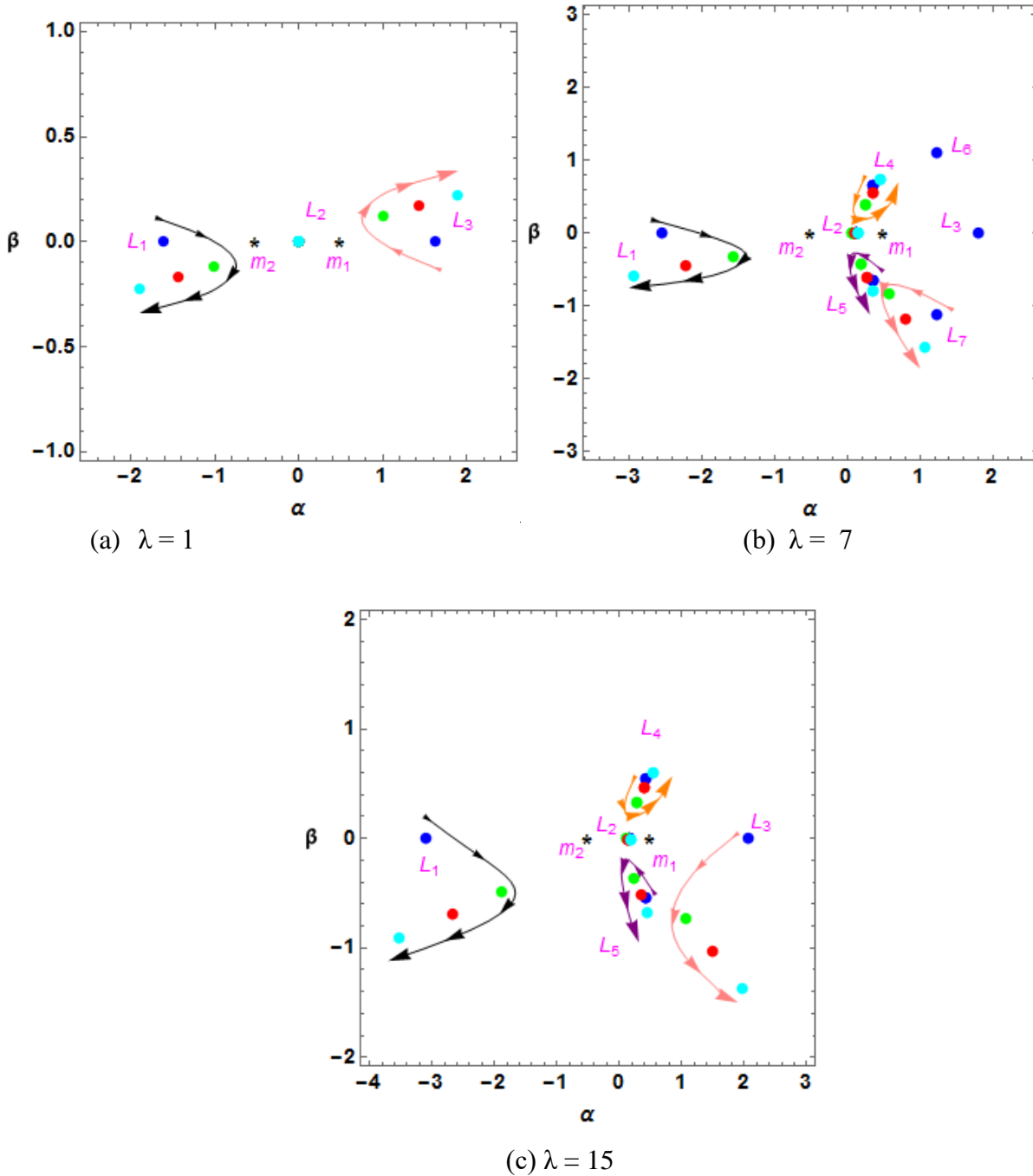(b) $\lambda = 7$

(c) $\lambda = 15$

Figure 5. Movement of equilibrium points at $\lambda_1 = 0$, $\lambda_3 = 1$ (Blue) and $\lambda_1 = 0.2$, $\lambda_3 = 0.4$ (Green), 0.8 (Red), 1.4 (Cyan) in $\alpha \beta$-plane.

Figure 2 presents the locations of equilibrium points at $\lambda = 1$ where figure 2(a) presents the classical case ($\lambda_1 = 0$ and $\lambda_3 = 1$) which coincides with [5], where three equilibrium points ($L_{1, 2, 3}$) were also obtained i.e. only collinear equilibrium points. Out of these equilibrium points L_2 is situated at the origin, which is also the center of mass of the primaries, while L_1 and L_3 lie left and right of the origin. Further, figures 2(b), 2(c) and 2(d) present the locations of equilibrium points when $\lambda_1 = 0.2$, $\lambda_3 = 0.4$, 0.8 and 1.4, respectively i.e. the variation of mass cases. In these three cases we observe that at $\lambda_1 = 0.2$ and $\lambda_3 = 0.4$, the two equilibrium points (L_1 and L_3) move toward the origin as well as moving anti-clockwise from the α-axis. As we increase the value of λ_3, we find that these two equilibrium points move away from the origin keeping L_2 at the origin. We can see the movement of the equilibrium points (L_1 and L_3) in figure 5(a).

Again, figure 3 presents the locations of equilibrium points at $\lambda = 7$, where figure 3(a) presents the classical case which also coincides with [5], where seven equilibrium points were obtained. But as we increase the value of λ_3 by

keeping the value of λ_1 the same at 0.2, we find only five equilibrium points, which mean that due to variation of mass, the number of equilibrium points reduces to five. The movement of all equilibrium points due to change of λ_3 can be seen in figure 5(b).

In the same way, figure 4 represents the case when $\lambda = 15$, where figure 4(a) represents the classical case and shows five equilibrium points, out of which three (L_1, L_2 and L_3) are collinear and two (L_4 and L_5) are non-collinear. But when we consider the variation of mass case, i.e. $\lambda_1 = 0.2$, $\lambda_3 = 0.4$, 0.8 and 1.4, we observe that L_1 and L_3 move downward from the α--axis. We can see the complete movement of these equilibrium points in figure 5(c). In this way, the variation parameters are working as catalytic agents.

3.2 Basins of the Attracting Domain

To solve the multivariate functions, we will use a simple, fast and accurate N-R iterative method. Therefore we illustrate the basins of the attracting domain for our present model. This method is activated when an initial condition is given to the configuration plane, while it stops when the positions of the equilibrium points are reached with some predefined accuracy. The regions of convergence are composed of all the initial values that tend to a specific equilibrium point. This is one of the most important qualitative properties of the dynamical systems. It is illustrated in the following way:
After classifying a dense uniform grid of 1024×1024 initial conditions, a multiple scan of the configuration plane was done.

By setting the maximum number of iterations to be 500, we set the predefined accuracy as 10^{-15}.
Using the above iterative method, we plotted the basins of attracting domain for three cases. The mathematical formulae for our model in α-β-plane when $\gamma = 0$, is presented by:

$$\alpha_{n+1} = \alpha_n - \left(\frac{(V_\alpha + V_1)(V_{\beta\beta} + V_{2\beta}) - (V_\beta + V_2)(V_{\alpha\beta} + V_{1\beta})}{(V_{\alpha\alpha} + V_{1\alpha})(V_{\beta\beta} + V_{2\beta}) - (V_{\alpha\beta} + V_{1\beta})(V_{\beta\alpha} + V_{2\alpha})} \right)_{(\alpha_n, \beta_n)}, \tag{17}$$

$$\beta_{n+1} = \beta_n - \left(\frac{(V_\beta + V_2)(V_{\alpha\alpha} + V_{1\alpha}) - (V_\alpha + V_1)(V_{\beta\alpha} + V_{2\alpha})}{(V_{\alpha\alpha} + V_{1\alpha})(V_{\beta\beta} + V_{2\beta}) - (V_{\alpha\beta} + V_{1\beta})(V_{\beta\alpha} + V_{2\alpha})} \right)_{(\alpha_n, \beta_n)}. \tag{18}$$

where α_n, β_n are the values of α and β coordinates of the nth step of the Newton-Raphson iterative process in Equations (17) and (18). The first and second derivatives are obtained from the right hand sides of the equations of motion (10). If the initial point converges rapidly to one of the equilibrium points then this point (α, β) will be a member of the basin of attracting domain. This process stops when the successive approximation converges to an attractor (in the dynamical system attractor means equilibrium point). For the classification of different equilibrium points on the plane, we used a color code.

Here we have represented the basins of attracting domain at $\lambda = 1$, 7, and 15 (taken from [5]) and these are shown in figures 6(a), 7(a) and 8(a). Figures 6(b), 7(b) and 8(b) are the zoomed parts of figures 6(a), 7(a) and 8(a) respectively. From figure 6(b), we observe that all the equilibrium points belong to cyan colored regions which extend to infinity. From figure 7(b), we observe that there are seven attracting points (equilibrium points) out of which L_1, L_4, L_5 and L_7 belong to the cyan, blue, light green and green colored regions which are extended to infinity respectively while L_2 belongs to the light blue colored region which covers a finite region. Again from figure 8(b), we observe that L_1 belongs to the cyan colored region, L_2, L_4 and L_5 belong to the light blue colored region, while L_3 belongs to the light green region, all of these regions extending to infinity.
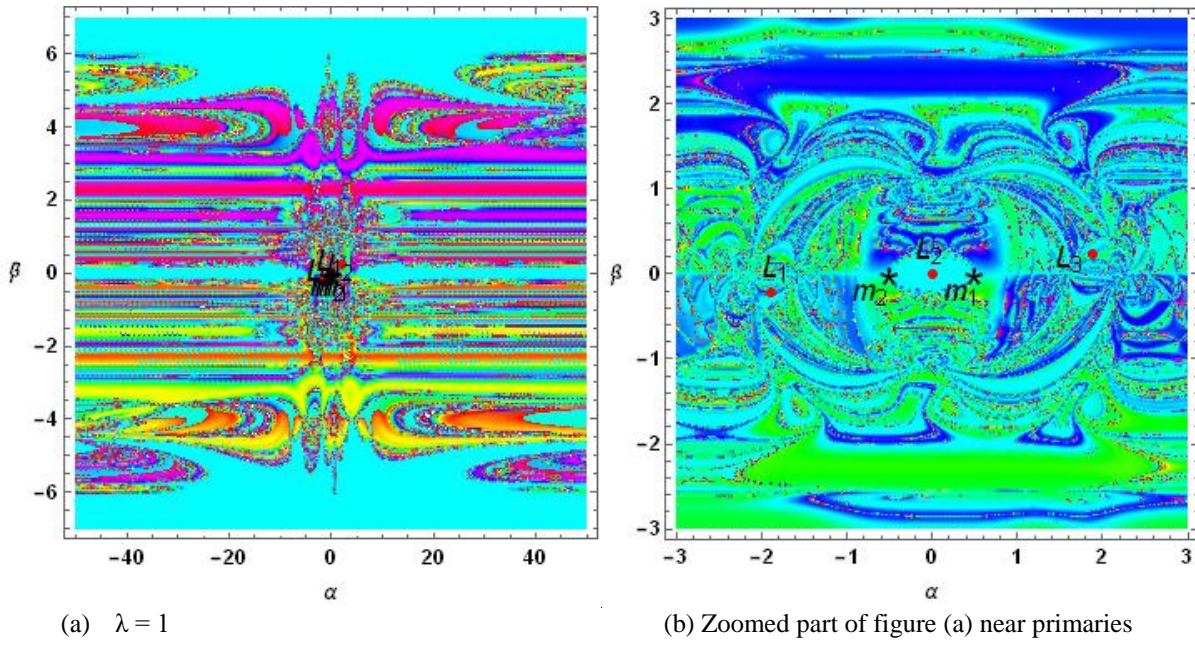
(a) $\lambda = 1$

(b) Zoomed part of figure (a) near primaries

Figure 6. Basins of attracting domain at $\lambda_1 = 0.2$, $\lambda_3 = 1.4$ in α β-plane.

(a) $\lambda = 7$

(b) Zoomed part of figure (a) near primaries
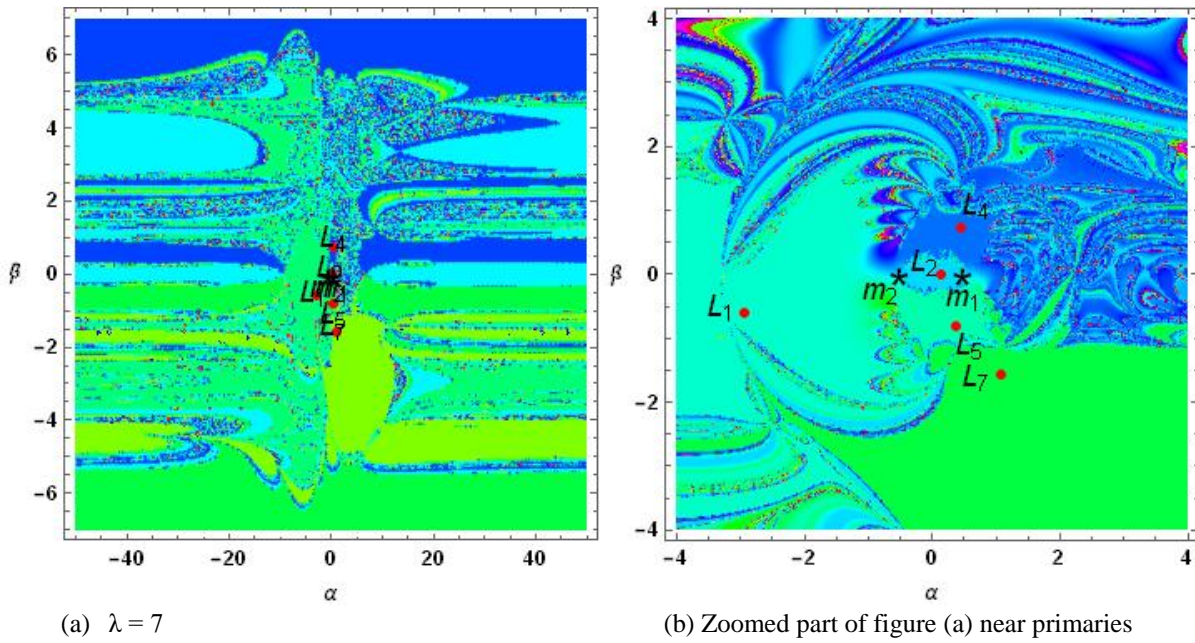
Figure 7. Basins of attracting domain at $\lambda_1 = 0.2$, $\lambda_3 = 1.4$ in α β-plane.

(a) $\lambda = 15$

(b) Zoomed part of figure (a) near primaries
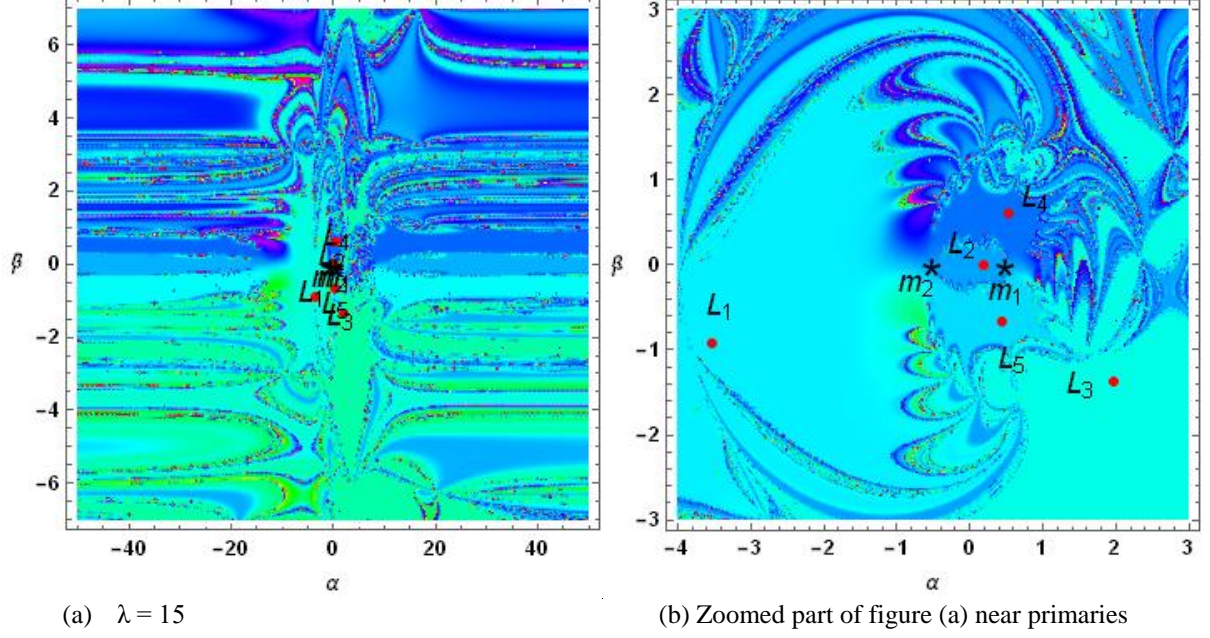
Figure 8. Basins of attracting domain at $\lambda_1 = 0.2$, $\lambda_3 = 1.4$ in $\alpha\,\beta$-plane.

4. Stability of Equilibrium Points

To reveal the stability properties of the small body's motion in its vicinity $(\alpha_0 + \alpha_1,\ \beta_0 + \beta_1,\ \gamma_0 + \gamma_1)$, where $\alpha_1,\ \beta_1,\ \gamma_1$ are small movements from the equilibrium points $(\alpha_0,\ \beta_0,\ \gamma_0)$. The variational equations for the system (10) can be written as:

$$\ddot{\alpha}_1 - f_3\dot{\beta}_1 + g_3\dot{\gamma}_1 = \left(V^0_{\alpha_1\alpha_1} + V^0_{1\alpha_1}\right)\alpha_1 + \left(V^0_{\alpha_1\beta_1} + V^0_{1\beta_1}\right)\beta_1 + \left(V^0_{\alpha_1\gamma_1} + V^0_{1\gamma_1}\right)\gamma_1,$$

$$\ddot{\beta}_1 - h_3\dot{\gamma}_1 + f_3\dot{\alpha}_1 = \left(V^0_{\beta_1\alpha_1} + V^0_{2\alpha_1}\right)\alpha_1 + \left(V^0_{\beta_1\beta_1} + V^0_{2\beta_1}\right)\beta_1 + \left(V^0_{\beta_1\gamma_1} + V^0_{2\gamma_1}\right)\gamma_1,$$

$$\ddot{\gamma}_1 - g_3\dot{\alpha}_1 + h_3\dot{\beta}_1 = \left(V^0_{\gamma_1\alpha_1} + V^0_{3\alpha_1}\right)\alpha_1 + \left(V^0_{\gamma_1\beta_1} + V^0_{3\beta_1}\right)\beta_1 + \left(V^0_{\gamma_1\gamma_1} + V^0_{3\gamma_1}\right)\gamma_1. \qquad (19)$$

where the superscript 0 denotes the value at the corresponding equilibrium point.

In the phase space, the above system (19) can be rewritten as:

$$\dot{\alpha}_1 = \alpha_2,$$

$$\dot{\beta}_1 = \beta_2,$$

$$\dot{\gamma}_1 = \gamma_2,$$

$$\dot{\alpha}_2 = \left(V^0_{\alpha_1\alpha_1} + V^0_{1\alpha_1}\right)\alpha_1 + \left(V^0_{\alpha_1\beta_1} + V^0_{1\beta_1}\right)\beta_1 + \left(V^0_{\alpha_1\gamma_1} + V^0_{1\gamma_1}\right)\gamma_1 + f_3\beta_2 - g_3\gamma_2,$$

$$\dot{\beta}_2 = \left(V^0_{\beta_1\alpha_1} + V^0_{2\alpha_1}\right)\alpha_1 + \left(V^0_{\beta_1\beta_1} + V^0_{2\beta_1}\right)\beta_1 + \left(V^0_{\beta_1\gamma_1} + V^0_{2\gamma_1}\right)\gamma_1 + h_3\gamma_2 - f_3\alpha_2,$$

$$\dot{\gamma}_2 = \left(V^0_{\gamma_1\alpha_1} + V^0_{3\alpha_1}\right)\alpha_1 + \left(V^0_{\gamma_1\beta_1} + V^0_{3\beta_1}\right)\beta_1 + \left(V^0_{\gamma_1\gamma_1} + V^0_{3\gamma_1}\right)\gamma_1 + g_3\alpha_2 - h_3\beta_2. \tag{20}$$

We use Meshcherskii space-time inverse transformations to examine the stability of the equilibrium points because the mass and distance of the small particle change with time.

$$\alpha_3 = \lambda_3^{-1/2}\alpha_1, \ \beta_3 = \lambda_3^{-1/2}\beta_1, \ \gamma_3 = \lambda_3^{-1/2}\gamma_1,$$

$$\alpha_4 = \lambda_3^{-1/2}\alpha_2, \ \beta_4 = \lambda_3^{-1/2}\beta_2, \ \gamma_4 = \lambda_3^{-1/2}\gamma_2. \tag{21}$$

With the help of equation (21), the system (20) can be written as follows:

$$\dot{X} = M\,X,$$

where

$$\dot{X} = \begin{pmatrix} \dot{\alpha}_3 \\ \dot{\beta}_3 \\ \dot{\gamma}_3 \\ \dot{\alpha}_4 \\ \dot{\beta}_4 \\ \dot{\gamma}_4 \end{pmatrix}, \quad X = \begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \\ \alpha_4 \\ \beta_4 \\ \gamma_4 \end{pmatrix} \text{ and}$$

$$M = \begin{pmatrix} \frac{1}{2}\lambda_1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2}\lambda_1 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2}\lambda_1 & 0 & 0 & 1 \\ V^0_{\alpha_1\alpha_1} + V^0_{1\alpha_1} & V^0_{\alpha_1\beta_1} + V^0_{1\beta_1} & V^0_{\alpha_1\gamma_1} + V^0_{1\gamma_1} & \frac{1}{2}\lambda_1 & f_3 & -g_3 \\ V^0_{\beta_1\alpha_1} + V^0_{2\alpha_1} & V^0_{\beta_1\beta_1} + V^0_{2\beta_1} & V^0_{\beta_1\gamma_1} + V^0_{2\gamma_1} & -f_3 & \frac{1}{2}\lambda_1 & h_3 \\ V^0_{\gamma_1\alpha_1} + V^0_{3\alpha_1} & V^0_{\gamma_1\beta_1} + V^0_{3\beta_1} & V^0_{\gamma_1\gamma_1} + V^0_{3\gamma_1} & g_3 & -h_3 & \frac{1}{2}\lambda_1 \end{pmatrix}.$$

The characteristic equation for the matrix M is

$$\lambda_4^6 + \alpha_5\lambda_4^5 + \alpha_4\lambda_4^4 + \alpha_3\lambda_4^3 + \alpha_2\lambda_4^2 + \alpha_1\lambda_4 + \alpha_0 = 0, \tag{22}$$

where

$$\alpha_0 = \frac{1}{64}\left(-8\left(2\left(-4\left(V^0_{1\gamma_1} + V^0_{\alpha_1\gamma_1}\right)\left(V^0_{2\beta_1} + V^0_{\beta_1\beta_1}\right) + 4\left(V^0_{1\beta_1} + V^0_{\alpha_1\beta_1}\right)\left(V^0_{2\gamma_1} + V^0_{\beta_1\gamma_1}\right)\right)\right)\right.$$

$$\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right) - 2(-4\begin{pmatrix}\left(V^0_{1\gamma_1} + V^0_{\alpha_1\gamma_1}\right)\left(V^0_{2\alpha_1} + V^0_{\beta_1\alpha_1}\right) \\ +4\left(V^0_{1\alpha_1} + V^0_{\alpha_1\alpha_1}\right)\left(V^0_{2\gamma_1} + V^0_{\beta_1\gamma_1}\right)\end{pmatrix}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)$$

$$+2\left(-4\left(V^0_{1\beta_1} + V^0_{\alpha_1\beta_1}\right)\left(V^0_{2\alpha_1} + V^0_{\beta_1\alpha_1}\right) + 4\left(V^0_{1\alpha_1} + V^0_{\alpha_1\alpha_1}\right)\left(V^0_{2\beta_1} + V^0_{\beta_1\beta_1}\right)\right)\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\right)$$

$$+8h_3\left(-4\left(V^0_{1\gamma_1} + V^0_{\alpha_1\gamma_1}\right)\left(V^0_{2\alpha_1} + V^0_{\beta_1\alpha_1}\right) + 4\left(V^0_{1\alpha_1} + V^0_{\alpha_1\alpha_1}\right)\left(V^0_{2\gamma_1} + V^0_{\beta_1\gamma_1}\right)\right)\lambda_1\right)$$

$$+8g_3\left(-4\left(V^0_{1\gamma_1} + V^0_{\alpha_1\gamma_1}\right)\left(V^0_{2\beta_1} + V^0_{\beta_1\beta_1}\right) + 4\left(V^0_{1\beta_1} + V^0_{\alpha_1\beta_1}\right)\left(V^0_{2\gamma_1} + V^0_{\beta_1\gamma_1}\right)\right)\lambda_1\right)$$

$$+ 32f_3V^0_{2\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1 - 32f_3V^0_{\beta_1\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1$$

$$+ 8\left(4h_3V^0_{1\beta_1} + 4g_3V^0_{1\beta_1} + 4h_3V^0_{\alpha_1\beta_1} + 4g_3V^0_{\beta_1\beta_1}\right)\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1$$

$$+ 32f_3V^0_{\beta_1\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1 - 32f_3V^0_{1\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1$$

$$- 32f_3V^0_{\alpha_1\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1 + 32f_3V^0_{1\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1$$

$$- 8\left(4h_3V^0_{1\alpha_1} + 4g_3V^0_{2\alpha_1} + 4h_3V^0_{\alpha_1\alpha_1} + 4g_3V^0_{\beta_1\alpha_1}\right)\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1$$

$$- 32f_3V^0_{2\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1 + 32f_3V^0_{\alpha_1\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1 - 16f_3h_3V^0_{1\gamma_1}\lambda_1^2$$

$$- 16f_3g_3V^0_{2\gamma_1}\lambda_1^2 - 16f_3^2V^0_{3\gamma_1}\lambda_1^2 - 16f_3h_3V^0_{\alpha_1\gamma_1}\lambda_1^2 - 4h_3(4h_3V^0_{1\alpha_1} + 4g_3V^0_{2\alpha_1}$$

$$+ 4h_3V^0_{\alpha_1\alpha_1} + 4g_3V^0_{\beta_1\alpha_1})\lambda_1^2 - 4g_3(4h_3V^0_{1\beta_1} + 4g_3V^0_{2\beta_1} + 4h_3V^0_{\alpha_1\beta_1} + 4g_3V^0_{\beta_1\beta_1})\lambda_1^2$$

$$+4(-4\left(V^0_{1\beta_1} + V^0_{\alpha_1\beta_1}\right)\left(V^0_{2\alpha_1} + V^0_{\beta_1\alpha_1}\right) + 4\left(V^0_{1\alpha_1} + V^0_{\alpha_1\alpha_1}\right)\left(V^0_{2\beta_1} + V^0_{\beta_1\beta_1}\right))\lambda_1^2$$

$$- 16f_3g_3V^0_{\beta_1\gamma_1}\lambda_1^2 - 16f_3h_3\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1^2 - 16V^0_{1\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1^2$$

$$- 16V^0_{\alpha_1\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1^2 - 16f_3g_3\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1^2 - 16V^0_{2\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1^2$$

$$- 16V^0_{\beta_1\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1^2 - 16f_3^2V^0_{\gamma_1\gamma_1}\lambda_1^2 + 16V^0_{1\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1^2$$

$$+ 16V^0_{2\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1^2 + 16V^0_{\alpha_1\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1^2 + 16V^0_{\beta_1\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1^2$$

$$- 8f_3V^0_{1\beta_1}\lambda_1^3 + 8g_3V^0_{1\gamma_1}\lambda_1^3 + 8f_3V^0_{2\alpha_1}\lambda_1^3 - 8h_3V^0_{2\gamma_1}\lambda_1^3 - 8h_3V^0_{\beta_1\gamma_1}\lambda_1^3 - 8f_3V^0_{\alpha_1\beta_1}\lambda_1^3$$

$$+ 8g_3V^0_{\alpha_1\gamma_1}\lambda_1^3 + 8f_3V^0_{\beta_1\alpha_1}\lambda_1^3 - 8g_3\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1^3 + 8h_3\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1^3$$

$$+ 4f_3^2\lambda_1^4 + 4g_3^2\lambda_1^4 + 4h_3^2\lambda_1^4 - 4V^0_{1\alpha_1}\lambda_1^4 - 4V^0_{2\beta_1}\lambda_1^4 - 4V^0_{3\gamma_1}\lambda_1^4 - 4V^0_{\alpha_1\alpha_1}\lambda_1^4$$

$$- 4V^0_{\beta_1\beta_1}\lambda_1^4 - 4V^0_{\gamma_1\gamma_1}\lambda_1^4 + \lambda_1^6)$$

$$\alpha_1 = \frac{1}{64}\left(-16h_3\left(-4\left(V^0_{1\gamma_1} + V^0_{\alpha_1\gamma_1}\right)\left(V^0_{2\alpha_1} + V^0_{\beta_1\alpha_1}\right) + 4\left(V^0_{1\alpha_1} + V^0_{\alpha_1\alpha_1}\right)\left(V^0_{2\gamma_1} + V^0_{\beta_1\gamma_1}\right)\right)\right.$$

$$- 16g_3\left(-4\left(V^0_{1\gamma_1} + V^0_{\alpha_1\gamma_1}\right)\left(V^0_{2\beta_1} + V^0_{\beta_1\beta_1}\right) - 64f_3 V^0_{2\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)$$

$$+ 4\left(V^0_{1\beta_1} + V^0_{\alpha_1\beta_1}\right)\left(V^0_{2\gamma_1} + V^0_{\beta_1\gamma_1}\right)\right) + 64f_3 g_3 V^0_{2\gamma_1}\lambda_1$$

$$- 16\left(4h_3 V^0_{1\beta_1} + 4g_3 V^0_{2\beta_1} + 4h_3 V^0_{\alpha_1\beta_1} + 4g_3 V^0_{\beta_1\beta_1}\right)\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)$$

$$- 64f_3 V^0_{\beta_1\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right) + 64f_3 V^0_{1\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)$$

$$+ 64f_3 V^0_{\alpha_1\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right) + 64f_3 f_3 V^0_{3\gamma_1}\lambda_1 + 64f_3 h_3 V^0_{\alpha_1\gamma_1}\lambda_1$$

$$+ 16\left(4h_3 V^0_{1\alpha_1} + 4g_3 V^0_{2\alpha_1} + 4h_3 V^0_{\alpha_1\alpha_1} + 4g_3 V^0_{\beta_1\alpha_1}\right)\right)\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)$$

$$-64f_3 V^0_{1\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right) + 64f_3 V^0_{2\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)$$

$$- 64f_3 V^0_{\alpha_1\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right) + 64f_3 V^0_{\beta_1\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right) + 64f_3 h_3 V^0_{1\gamma_1}\lambda_1$$

$$+ 16h_3\left(4h_3 V^0_{1\alpha_1} + 4g_3 V^0_{2\alpha_1} + 4h_3 V^0_{\alpha_1\alpha_1} + 4g_3 V^0_{\beta_1\alpha_1}\right)\lambda_1$$

$$+ 16g_3\left(4h_3 V^0_{1\beta_1} + 4g_3 V^0_{2\beta_1} + 4h_3 V^0_{\alpha_1\beta_1} + 4g_3 V^0_{\beta_1\beta_1}\right)\lambda_1$$

$$-16\left(-4\left(V^0_{1\beta_1} + V^0_{\alpha_1\beta_1}\right)\left(V^0_{2\alpha_1} + V^0_{\beta_1\alpha_1}\right) + 4\left(V^0_{1\alpha_1} + V^0_{\alpha_1\alpha_1}\right)\left(V^0_{2\beta_1} + V^0_{\beta_1\beta_1}\right)\right)\lambda_1$$

$$+ 64f_3 h_3\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1 + 64f_3 g_3 V^0_{\beta_1\gamma_1}\lambda_1 + 64V^0_{1\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1$$

$$+ 64V^0_{2\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1 + 64V^0_{\alpha_1\gamma_1}\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1$$

$$+ 64f_3 g_3\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1 + 64V^0_{\beta_1\gamma_1}\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1 + 64f_3 f_3 V^0_{\gamma_1\gamma_1}\lambda_1$$

$$- 64V^0_{1\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1 - 64V^0_{2\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1$$

$$- 64V^0_{\alpha_1\alpha_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1 + 64V^0_{\beta_1\beta_1}\left(V^0_{3\gamma_1} + V^0_{\gamma_1\gamma_1}\right)\lambda_1\right) + 48f_3 V^0_{1\beta_1}\lambda_1^2$$

$$- 48g_3 V^0_{1\gamma_1}\lambda_1^2 - 48f_3 V^0_{2\alpha_1}\lambda_1^2 + 48h_3 V^0_{2\gamma_1}\lambda_1^2 + 48f_3 V^0_{\alpha_1\beta_1}\lambda_1^2 - 48g_3 V^0_{\alpha_1\gamma_1}\lambda_1^2$$

$$- 48f_3 V^0_{\beta_1\alpha_1}\lambda_1^2 + 48h_3 V^0_{\beta_1\gamma_1}\lambda_1^2 + 48g_3\left(V^0_{3\alpha_1} + V^0_{\gamma_1\alpha_1}\right)\lambda_1^2$$

$$- 48h_3\left(V^0_{3\beta_1} + V^0_{\gamma_1\beta_1}\right)\lambda_1^2 - 32f_3^2\lambda_1^3 - 32g_3^2\lambda_1^3 - 32h_3^2\lambda_1^3 + 32V^0_{1\alpha_1}\lambda_1^3$$

$$+ 32V^0_{2\beta_1}\lambda_1^3 + 32V^0_{3\gamma_1}\lambda_1^3 + 32V^0_{\alpha_1\alpha_1}\lambda_1^3 + 32V^0_{\beta_1\beta_1}\lambda_1^3 + 32V^0_{\gamma_1\gamma_1}\lambda_1^3 - 12\lambda_1^5\Big)$$

$$\alpha_2 = \frac{1}{64}\left(-64f_3 h_3 V^0_{1\gamma_1} - 64f_3 g_3 V^0_{2\gamma_1} - 64f_3 h_3 V^0_{1\gamma_1} - 64f_3 f_3 V^0_{3\gamma_1} - 64f_3 h_3 V^0_{\alpha_1\gamma_1}\right.$$

$$- 16h_3\left(4h_3 V^0_{1\alpha_1} + 4g_3 V^0_{2\alpha_1} + 4h_3 V^0_{\alpha_1\alpha_1} + 4g_3 V^0_{\beta_1\alpha_1}\right)$$

$$- 16g_3\left(4h_3 V^0_{1\beta_1} + 4g_3 V^0_{2\beta_1} + 4h_3 V^0_{\alpha_1\beta_1} + 4g_3 V^0_{\beta_1\beta_1}\right) - 64f_3 g_3 V^0_{\beta_1\gamma_1}$$

$$+16\left(-4\left(V^0_{1\beta_1}+V^0_{\alpha_1\beta_1}\right)\left(V^0_{2\alpha_1}+V^0_{\beta_1\alpha_1}\right)+4\left(V^0_{1\alpha_1}+V^0_{\alpha_1\alpha_1}\right)\left(V^0_{2\beta_1}+V^0_{\beta_1\beta_1}\right)\right)$$

$$-64f_3f_3V^0_{\gamma_1\gamma_1}-64f_3h_3\left(V^0_{3\alpha_1}+V^0_{\gamma_1\alpha_1}\right)-64V^0_{1\gamma_1}\left(V^0_{3\alpha_1}+V^0_{\gamma_1\alpha_1}\right)$$

$$-64V^0_{\alpha_1\gamma_1}\left(V^0_{3\alpha_1}+V^0_{\gamma_1\alpha_1}\right)-64f_3g_3\left(V^0_{3\beta_1}+V^0_{\gamma_1\beta_1}\right)-64V^0_{2\gamma_1}\left(V^0_{3\beta_1}+V^0_{\gamma_1\beta_1}\right)$$

$$-64V^0_{\beta_1\gamma_1}\left(V^0_{3\beta_1}+V^0_{\gamma_1\beta_1}\right)+64V^0_{1\alpha_1}\left(V^0_{3\gamma_1}+V^0_{\gamma_1\gamma_1}\right)+64V^0_{2\beta_1}\left(V^0_{3\gamma_1}+V^0_{\gamma_1\gamma_1}\right)$$

$$+64V^0_{\alpha_1\alpha_1}\left(V^0_{3\gamma_1}+V^0_{\gamma_1\gamma_1}\right)+64V^0_{\beta_1\beta_1}\left(V^0_{3\gamma_1}+V^0_{\gamma_1\gamma_1}\right)-96f_3V^0_{1\beta_1}\lambda_1$$

$$+96g_3V^0_{1\gamma_1}\lambda_1+96f_3V^0_{2\alpha_1}\lambda_1-96f_3V^0_{\alpha_1\beta_1}\lambda_1+96g_3V^0_{\alpha_1\gamma_1}\lambda_1+96f_3V^0_{\beta_1\alpha_1}\lambda_1$$

$$-96h_3V^0_{\beta_1\gamma_1}\lambda_1-96h_3V^0_{2\gamma_1}\lambda_1-96g_3\left(V^0_{3\alpha_1}+V^0_{\gamma_1\alpha_1}\right)\lambda_1$$

$$+96h_3\left(V^0_{3\beta_1}+V^0_{\gamma_1\beta_1}\right)\lambda_1+96f_3^2\lambda_1^2+96g_3^2\lambda_1^2+96h_3^2\lambda_1^2-96V^0_{1\alpha_1}\lambda_1^2$$

$$-96V^0_{2\beta_1}\lambda_1^2-96V^0_{3\gamma_1}\lambda_1^2-96V^0_{\alpha_1\alpha_1}\lambda_1^2-96V^0_{\beta_1\beta_1}\lambda_1^2-96V^0_{\gamma_1\gamma_1}\lambda_1^2+60\lambda_1^4\Big),$$

$$\alpha_3=f_3V^0_{1\beta_1}-g_3V^0_{1\gamma_1}-f_3V^0_{2\alpha_1}+h_3V^0_{2\gamma_1}+f_3V^0_{\alpha_1\beta_1}-g_3V^0_{\alpha_1\gamma_1}-f_3V^0_{\beta_1\alpha_1}+h_3V^0_{\beta_1\gamma_1}$$

$$+2V^0_{2\beta_1}\lambda_1+g_3\left(V^0_{3\alpha_1}+V^0_{\gamma_1\alpha_1}\right)-h_3\left(V^0_{3\beta_1}+V^0_{\gamma_1\beta_1}\right)-2f_3^2\lambda_1-2g_3^2-2h_3^2\lambda_1$$

$$+2V^0_{1\alpha_1}\lambda_1+2V^0_{\beta_1\beta_1}\lambda_1+2V^0_{\alpha_1\alpha_1}\lambda_1+2V^0_{\gamma_1\gamma_1}\lambda_1-\frac{5}{2}\lambda_1^3,$$

$$\alpha_4=f_3^2+g_3^2+h_3^2-V^0_{1\alpha_1}-V^0_{2\beta_1}-V^0_{3\gamma_1}-V^0_{\alpha_1\alpha_1}-V^0_{\beta_1\beta_1}-V^0_{\gamma_1\gamma_1}+\frac{15}{4}\lambda_1^2,$$

$$\alpha_5=-3\lambda_1.$$

We have solved equation (22) numerically for three different values of parameter λ (1, 7, 15) and evaluated characteristic roots which are given in tables (1-3). We observed from these roots given in these three tables 1, 2, 3 that all the equilibrium points are unstable because at-least one characteristic root is either a positive real number or positive real part of the complex characteristic root, while [5] found in his investigation that some equilibrium points are stable in some interval values for the parameter λ. In this way we can say that these variation parameters convert to all the equilibrium points as unstable. Therefore it is easy to say that these variation parameters have great impact on this dynamical system.

Table 1. Corresponding characteristic roots of equilibrium points in α-β-plane at λ = 1, λ_1 = 0.2, λ_3 = 1.4.

S.N.	Equilibrium Point (α, β)	Roots	Nature
1	1.8922290573, - 0.2249047710	+ 0.1238455712 ± 0.9648772652 i + 1.8829013676, - 1.7305925101 + 0.1000000000, + 0.1000000000	Unstable
2	0.0000000000, 0.0000000000	+ 0.1919166735 ± 15.3247492922 i + 2.9831536654, + 0.0999999999 - 2.9669870125, + 0.1000000000	Unstable
3	1.8922290573, 0.2249047710	+ 0.1238455712 ± 0.9648772652 i + 1.8829013676, + 0.1000000000 + 0.0999999999, - 1.7305925101	Unstable

Table 2. Corresponding characteristic roots of equilibrium points in α-β-plane at λ = 7, λ₁ = 0.2, λ₃ = 1.4.

S.N.	Equilibrium Point (α, β)	Roots	Nature
1	- 2.9288319681, - 0.5956468434	+ 0.1244710980 ± 0.7705292447 i + 1.7304695528, - 1.5794117488 + 0.1000000000, + 0.1000000000	Unstable
2	0.1420900714, - 0.0070146815	+ 0.1993522521 ± 46.6873463284 i + 2.8958790218, + 0.1000000000 - 2.8945835262, + 0.0999999999	Unstable
3	0.4622777422, 0.7320519601	+ 0.0897598108 ± 7.5033419383 i + 1.8018965165, + 0.0999999999 + 0.1000000000, - 1.5814161383	Unstable
4	0.3598259379, - 0.8004678161	+ 0.0957408351 ± 6.8651530569 i + 0.1000000000, + 1.6062369651 - 1.3977186353, + 0.1000000000	Unstable
5	1.0704385208, - 1.5676578978	+ 1.2156733907 ± 0.0888414341 i + 0.1000000000, - 0.8781507268 - 1.1531960546, + 0.1000000000	Unstable

Table 3. Corresponding characteristic roots of equilibrium points in α-β-plane at λ = 15, λ₁ = 0.2, λ₃ = 1.4.

S.N.	Equilibrium Point (α, β)	Roots	Nature
1	- 3.5192043182, - 0.9124002233	+ 0.1245222506 ± 0.6945789131 i + 1.6854000078, + 0.0999999999 - 1.5344445091, + 0.1000000000	Unstable
2	+ 0.1939056387, - 0.0096053140	+ 0.1999647710 ± 76.6270430841 i + 2.8730485051, - 2.8729780473 + 0.1000000000, + 0.1000000000	Unstable
3	1.9795166567, - 1.3700857646	+ 0.1000000000, - 1.2570087540 - 0.5578040967, + 0.8068614037 + 1.4079514469, + 0.1000000000	Unstable
4	0.5347180082, 0.6049358108	+ 0.0922083409 ± 17.2608437186 i + 1.8078564632, + 0.1000000000 - 1.5922731451, + 0.1000000000	Unstable
5	- 0.6759400828, + 0.4504694723	+ 0.1005874123 ± 15.9957018321 i + 1.5269661797, - 1.3281410043 + 0.1000000000, + 0.1000000000	Unstable

Conclusion

In this paper we have investigated the effect of variation of mass of the infinitesimal body (test particle) under the influence of the electromagnetic Copenhagen problem. We have found that these variation parameters have great impact on the behavior of this dynamical system. In our investigation, firstly we have determined the equations of motion where both variation parameters λ_1 and λ_3 are clearly visible. Using this system of equations of motion we numerically illustrated the equilibrium points for three values of magnetic moment (λ = 1, 7, 15) (taken from [5]) and respectively we found three, seven and five equilibrium points for the four different values of variation parameters λ_1 = 0, 0.2 and λ_3 = 1, 0.4, 0.8, 1.4. The complete movements of these equilibrium points are presented in figure (5). We have also performed the Newton-Raphson basins of attracting domain in the above said three cases of λ and for fixed values of λ_1 = 0.2 and λ_3 = 1.4. The variations of attracting domain are presented in the figures 6, 7 and 8. For a clearer view, we have shown the figures in the zoomed parts of figures 6(a), 7(a) and 8(a) in figures 6(b), 7(b) and 8(b), respectively. In the following investigation we have explored numerically the stability of corresponding equilibrium points, for which we have evaluated the characteristic roots corresponding to the equilibrium points, and these are given in tables 1, 2 and 3. We observed from the tables that all the equilibrium points are unstable, which finding is clearly different from that of the investigation done by [5].

Conflict of Interest

The author declares no conflict of interest.

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