

# Equivalence of Electromagnetic Fluctuation and Nuclear (Yukawa) Forces: the $\pi_0$ Meson, its Mass and Lifetime

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## Abstract

It is shown how Maxwell's equations for the electromagnetic field with Planck quantisation of allowed modes appears to provide a semiclassical account of nuclear interactions. The mesons emerge as plasmons, collective excitations in an electron positron pair sea. The lifetime and mass of  $\pi_0$  mesons are predicted.

**Keywords:** Casimir-effect, Meson-theory, positron-electron-plasma, lifetime

### 1. Electrodynamic fluctuation forces.

Feynman is reported to have believed that there had to be a connection between electromagnetic theory and nuclear forces. [1]. He never found such a link. It is shown how such a connection might come about.

### Historical background: Where do mesons come from??

A hundred years ago Rutherford's team at Cambridge had shown that the atomic nucleus was comprised of protons and neutrons. The particles had a mass, 2000 times that of an electron ; protons were positively charged. A neutron could transform into a proton and a negatively charged electron. Electrostatic forces played a role in the interactions between nucleons. But whatever other forces held them together remained a mystery [2]. Quantum mechanics in its various manifestations, from Planck, Sommerfeld and Bohr, Schroedinger, Heisenberg, Dirac; and later quantum electrodynamics

promised insights. In 1935 Yukawa had come up with a characterisation of this so called “weak nuclear interaction” that worked. The force was mediated by “particles” called mesons, mass 273 times that of an electron and variously charged. They were detected from cosmic ray decay by Powell in 1937. There were  $\mu$  and  $\pi$  and later bigger  $K$  mesons. Particle physics developed subsequently culminating in the prediction of Higgs unifying boson. But the fundamental physics embodied in Maxwell’s equations for the electromagnetic field and quantum mechanics seemed to have nothing much to do with it. Somehow something was apparently missing. The electromagnetic forces seemed too small. But protons and electrons were charged. The mystery and disjunction remained.

### **Theory in words without equations**

The classic paper of Casimir in 1948 [3] on relativistic effects on the attractive forces between colloidal particles was motivated by, and applied to the newly developed Deryaguin-Overbeek theory of Colloid stability. Overbeek had posed the problem of these “retardation” effects to his friend Casimir in Utrecht (BWN private communication with Overbeek). Casimir derived the forces due to quantisation of zero temperature electromagnetic fluctuations in the vacuum between two ideal metal plates. It stimulated a huge literature that still flourishes. It seemed to bear on our problem. But it could not due to its limitation to zero temperature. This and the term retardation are incorrect and unphysical [4-6].

Further insights into the nature of the electromagnetic vacuum had to wait on the development of Lifshitz theory for interactions between and across dielectric media [7] and included temperature. This theory at time appeared to be the culmination and triumph of quantum electrodynamics. It had been foreshadowed by P. N. Lebedev who discovered light radiation

pressure. He was a friend of J. Clerk Maxwell and the stepfather of Deryaguin. Deryaguin had asked Lifshitz to work on the problem. In 1894 Lebedev wrote: "If the solution of this problem ever becomes possible we shall be able, from the results of spectral analysis, to calculate in advance the values of the intermolecular forces due to molecular inter-radiation, deduce the laws of their temperature dependence, and, by comparing the values obtained with experimental results, solve the fundamental problem of molecular physics whether all the so-called 'molecular forces' are confined to the already known mechanical action flight radiation mentioned above, to electromagnetic forces, or whether some forces of hitherto unknown origin are involved" as quoted by Deryaguin [8]. However the triumph was illusory. The generalisation of the Casimir effect involved some sleight of hand that approximated a non linear problem by a linear one [9].

This theory applied to Casimir's two plate problem gives out automatically: the binding energy of two nucleons in nucleus in equilibrium and automatically replaces the problem by one with a virtual intervening electron positron pair sea with known density and therefore plasma frequency of excitations. The renormalisation is identical to the Klein Gordon equation for scalar mesons with mass identified from the plasma frequency. The implication is that positive and negatively charged  $\pi$  mesons are identifiable with bound electron-plasma and positron-plasma excitations. And what used to be called  $K$  mesons are higher order double plasma excitations known from solid state physics. What is quite new is that the identification of the scalar  $\pi_0$  meson with a collective excitation in the electron positron sea allows us to calculate its lifetime, correctly. Taken together, binding energy, scalar meson mass, and lifetime all seem to add plausibility to our case. The simplified version of Lifshitz theory we have used is the same Lifshitz theory at the foundations of physical chemistry, molecular and colloidal particle interactions in the DLVO

theory. There the limitations due to the linearisation approximation are very clear. If the equivalence we have drawn is correct so too must present theories of particle physics.

**We first outline what we mean by electromagnetic forces.**

A 1961 paper of Dzyaloshinski, Lifshitz and Pitaevski [7] applied quantum electrodynamics to the problem of molecular forces. It extended earlier work on electromagnetic fluctuation forces between molecules and colloidal particles of Casimir and Lifshitz to include effects of an intervening medium between the interacting particles. This impressive advance turned out later to be flawed. An approximation made in the derivation meant that the formidable mathematical formalism collapsed to a semi-classical theory. By this we mean Maxwell's equations with boundary conditions and quantisation of allowed modes [6,9,10].

Technically the reason for this is that in the development of the theoretical formalism there occurs an integral equation for the polarisation operator that involves a non-linear coupling constant integration. An approximate solution can be found by linearising. The true polarization operator is then replaced by the macroscopic dielectric susceptibility. A detailed exposition can be found in Eq. 2.9 and Eq. 3.1 in Ref. [7].

## **2. Theory**

### **2.1 Model Assumptions and the Casimir Energy**

We assume that the nucleons have a structure which involves electromagnetic forces somehow as protons have a positive charge and a magnetic moment. Then, how much of a role could electromagnetic forces play in nuclear interactions? Consider two nucleons. If the nucleons were

perfectly reflecting spheres, calculation of the electromagnetic fluctuation forces would require an analytic solution of the Helmholtz equation. This is complicated [11,12]. So we simplify the model and approximate the nucleons by perfectly reflecting planes with the same cross sectional area as the (spherical) nucleons.

Then the attractive electromagnetic fluctuation energy of interaction (all energies in this work are given per unit area) across a vacuum at zero temperature is [3]

$$E = -\frac{\pi^2 \hbar c}{720 d^3}. \quad (1)$$

Here  $d$  is the distance between the plates,  $\hbar$  is Planck's constant and  $c$  the velocity of light. We take  $d$  to be the distance between surfaces of the protons. The effective surface area is  $A = \pi r^2$ ,  $r$ =proton radius $\sim$ 0.8 fermi. A typical nucleon-nucleon surface to surface distance is of the order of one fermi. Then the available two nucleon-nucleon energy for binding in a nucleus from vacuum fluctuations is about 5 MeV. The implication is that there is enough electromagnetic energy available in the zero-point Casimir energy to account for nuclear interactions. The binding energy per nucleon varies in different atomic nuclei but is typically in the range from 1.1 MeV to 8.8 MeV.

## 2.2 Temperature Dependence of Electromagnetic Forces

The observation that the zero temperature Casimir vacuum fluctuation energy is enough to provide the binding energy of nucleons in a nucleus is suggestive. To take matters further we need to consider the effects of temperature. The Gibbs free energy extension of Casimir's result that does so is due to Lifshitz, it is [7,9,10,13],

$$G(d, T) = \frac{kT}{\pi} \sum_{n=0}^{\infty} ' \int_0^{\infty} dq q \ln \left[ 1 - e^{-2d\sqrt{q^2 + \xi_n^2/c^2}} \right], \quad (2)$$

where  $k$  is Boltzmann's constant,  $T$  is temperature,  $q$  is the wavevector, and  $\xi_n = 2\pi n k T / \hbar$ . The prime indicates that the zero frequency term carries a factor of one half. Explicitly, at small distances, or high temperatures, this has the expansion [14],

$$G(d, T) \approx \frac{-\pi^2 \hbar c}{720 d^3} - \frac{\zeta(3) k^3 T^3}{2 \pi \hbar^2 c^2} + \frac{\pi^2 d k^4 T^4}{45 \hbar^3 c^3} + \dots, \quad (3)$$

$\zeta(3) \approx 1.202$  is a zeta function. Here the first term is the attractive (zero temperature) Casimir result. The third term is the equilibrium black body radiation energy in the vacuum between the plates. It opposes the attractive Casimir term. Additional exponentially decaying terms are negligible in the regime of interest and have been omitted. Leaving aside the second term for the moment, we suppose that the first and third terms are equal at equilibrium. This then provides us with a temperature determined by the distance  $d$  between the two plates,  $T = \frac{\hbar c}{2kd}$ , at which the attractive and repulsive forces balance.

### The Electron—Positron Sea

The second term, is a chemical potential term in the Gibbs free energy. We can recognise it explicitly as due to an electron positron pair sea formed from the photons in the gap by the reaction  $e^+ + e^- \leftrightarrow \gamma$  [11]. From the temperature at distance  $d$  we can calculate the density for this electron

positron pair sea. As discussed by Landau and Lifshitz [15] the number of electrons and positrons are very nearly identical and both very large, even at temperatures of the order of  $mc^2$ . (An electron-positron plasma becomes more nearly perfect with increasing density so we can use perfect gas formulae and ignore correlations .)

The second term can then be re-written as

$$\frac{\zeta(3) k^3 T^3}{2 \pi \hbar^2 c^2} = \frac{\pi(\rho_- + \rho_+) \hbar c}{6}, \quad (4)$$

where we use the expression for the density ( $\rho = \rho_- + \rho_+ = \frac{3\zeta(3)(kT)^3}{\pi^2 \hbar^3 c^3}$ ) of the electron-positron plasma [15]. The interpretation of the chemical potential term (the second term in Eq. (3)) is the key to the equivalence we seek.

### 2.3 Reformulation: the Klein Gordon Equation and Meson Mass

The imposition of a balance between the vacuum fluctuation and black body radiation forces has reformulated the problem to be that of an electromagnetic fluctuation force in which there are two metal plates separated by a medium. This medium, an electron-positron plasma, has the permittivity

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}; \quad \omega_p^2 = \frac{4\pi(\rho_- + \rho_+)e^2}{m_e}, \quad (5)$$

where  $e$  is the unit electric charge and  $m_e$  is the mass of the electron. The electromagnetic fluctuation interaction energy between two perfectly conducting plates across a plasma can be derived from the equation for the scalar potential, in Maxwell's equations [9], which after a Fourier transform reduces to,

$$\nabla^2 \phi + \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2}\right) \phi = 0, \quad (6)$$

Yukawa [16] proposed that the nuclear interaction could be derived from the Klein-Gordon equation,

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \mu^2\right) \phi_\pi = 0. \quad (7)$$

This equation has the solution  $\phi_\pi \sim \pm \frac{g^2 e^{-\mu d}}{d}$  [16]. The range of the Yukawa potential is inversely proportional to the meson mass ( $m_\pi$ ):  $d_\pi = \frac{1}{\mu} = \hbar/(m_\pi c)$ . One can proceed from this known relationship between  $d_\pi$  and  $m_\pi$  [2]. But, for later work on the lifetime it is useful to recall first the very basic physical assumptions used to relate the meson mass to the Yukawa decay length. As discussed by Wick [2], mesons act via emission and absorption processes of virtual excitations, and the time required for the excitation to travel between a pair of nucleons is of the order  $\Delta t \sim \frac{d_\pi}{c}$ . The relativistic energy,  $\Delta E \geq m_\pi c^2$ , obeys the Heisenberg uncertainty principle for energy [17]:  $\Delta E \Delta t \geq \hbar$ . These expressions for energy and time lead us to the required relationship:  $d_\pi \cong \hbar/(m_\pi c)$ .

The Klein-Gordon equation for Yukawa potential ( $\phi$ ) after a Fourier transform may be cast into the form [16],

$$\nabla^2 \phi_\pi + \frac{\omega^2}{c^2} \left( 1 - \frac{1}{\omega^2} \left( \frac{m_\pi c^2}{\hbar} \right)^2 \right) \phi_\pi = 0. \quad (8)$$

We identify this equation with equation (6). Thus, we obtain after identification of Eq. (6) with Eq.

(8)  $\omega_p^2 = \frac{4\pi\rho e^2}{m_e} = \frac{c^2}{d_\pi^2} = \left( \frac{m_\pi c^2}{\hbar} \right)^2$ . The meson mass follows as

$$m_\pi = \frac{2e\hbar}{c} \sqrt{\frac{\pi\rho}{m_e c^2}}. \quad (9)$$

This gives  $m_\pi = 267m_e$  in surprising agreement with the experimental result ( $264m_e$ ). We discuss this result further in section 3. In this scenario the charged  $\pi^-$  and  $\pi^+$  mesons would emerge as electron-plasmon and positron-plasmon bound states.

#### 2.4 Binding Energy of Nucleons Casimir-Lifshitz theory

Returning to the model of Sec. 2.1, we have for the interaction of two perfectly conducting plates across an intervening plasma [14,18], the Gibbs free energy

$$G(d, T) = \frac{kT}{\pi} \sum_{n=0}^{\infty} ' \int_0^{\infty} dq q \ln \left[ 1 - e^{-2d\sqrt{q^2 + \kappa^2 + \xi_n^2/c^2}} \right], \quad (10)$$

where  $\kappa = \omega_p/c$ . For high temperatures at fixed separation, or large separation at fixed temperature, it follows [19,20] it has an expansion of the form:

$$G(d, T) = -\frac{kT\kappa}{4\pi} \frac{e^{-2\kappa d}}{d} \left[ 1 + \frac{1}{2d\kappa} \right] - \frac{(kT)^2 e^{-2\eta d}}{\hbar c} \frac{e^{-\rho^* \eta d}}{d} + O(e^{-4\eta d}), \quad (11)$$

where  $\rho^* = \rho e^2 \hbar^2 / (\pi m_e k^2 T^2)$ ,  $\eta = \frac{2kT}{\hbar c}$  and  $\kappa$  is defined above.

Both the  $n=0$  and  $n>0$  terms behave similarly to the Yukawa potential [16]. Both provide a contribution to our model nuclear binding energy that agrees very well with the experimentally observed binding energy per nucleon. We will compare our theoretical results with the typical experimental results in Sec. 3.

## 2.5 Lifetime of Plasmons and Mesons

Our assumption is that at equilibrium the zero point fluctuation energies of the vacuum and the black body radiation energy cancel out. What is left are collective excitations, plasmons in the remaining electron-positron sea. These can be identified as pions. This allows us to estimate the lifetime of the  $\pi_0$  meson. The lifetime is that for the decay of a plasmon into two electron-positron pairs [21]. These can decay to produce two photons. The theory of collective electron excitations plasmons is known. The broadening ( $\Delta E$ ) of the plasmon peak and its lifetime ( $\tau \geq 1/\Delta E$ ) is known analytically and measured [22],

$$\Delta E \sim \frac{6\pi\varepsilon_F}{5\hbar} \left( \frac{q\pi}{q_F} \right)^2 \left( \frac{\hbar\omega_p}{2\varepsilon_F} \right)^3 \left[ 10 \ln(2) + 2 - 4.5 \frac{\hbar\omega_p}{2\varepsilon_F} + O \left( \frac{\hbar\omega_p}{2\varepsilon_F} \right)^2 \dots \right], \quad (12)$$

The entities involved are the Fermi energy ( $\varepsilon_F \propto \rho^{2/3}$ ), plasma frequency ( $\omega_p \propto \rho^{1/2}$ ), and Fermi wavevector ( $q_F \propto \rho^{1/3}$ ). These depend on density and (in our case) on the distance between the nucleons. The lifetime dependence upon the electron-positron plasma density can be deduced once we have a model for the neutral pi meson (plasmon) wave vector. In order to calculate the lifetime of the plasmon we need an estimate for the  $q_\pi$ -vector. We use the relationship between q-vector and energy. The relativistic energy of the plasmon excitation (meson with mass  $m_\pi$ ),  $E \sim m_\pi c^2$ , [2], is assumed spread into kinetic energy ( $\frac{\hbar^2 q_\pi^2}{2m_e}$ ) for each particle of two electron-positron pairs (in general not all energy turns into the kinetic energy of these particles). This leads to an order of magnitude estimate for the wave vector of the plasmon:  $q_\pi \leq c \sqrt{\frac{m_\pi m_e}{2}} / \hbar$ . As we have shown in Eq. (9) that  $m_\pi \propto \rho^{1/2}$ , the broadening and lifetime is apparently independent of electron-positron density (and independent of separation between nucleon pairs). A possibly better estimate subtracts off the relativistic energy for each of the particles created in the two electron-positron pairs from the relativistic energy of the plasmon. This leads to:  $q_\pi \leq c \sqrt{\frac{(m_\pi - 4m_e)m_e}{2}} / \hbar$ , with only a slight density dependence for the lifetime. The estimate will be seen to lead to the same numerical value (to the first decimal place) as the “naive” (Weinberg’s word [23]), QFT (Quantum Field Theory) approximation for the uncharged pion lifetime. Both our result for lifetime and the “naive” one have the same order of magnitude ( $\sim 0.2 \times 10^{-16}$ s). This can be compared with the state-of-the-art QFT result ( $0.80\text{-}0.85 \times 10^{-16}$ s) which agrees with the experimental value ( $0.834 \times 10^{-16}$ s), cf. Sec. 3.

### 3. Results

#### 3.1 Numerical, Experimental and selected QFT Results for $\pi_0$ Mesons

##### Meson Mass

The equivalent black box at a nucleon pair separation of 1 fermi or closer contains very nearly the maximum number of electron positron pairs. If we take  $d \sim 1.5$  fermi, the equivalent temperature is  $kT \sim 128m_e c^2$ . This leads via Eq. (9) to a meson mass of  $267m_e$  which compares remarkably favorably with the experimental results [24,25],  $m_e \approx 0.511 \text{ MeV}$  and  $m_\pi \approx 134.97 \text{ MeV} \approx 264m_e$ . The dependence of the estimated meson mass on nucleon separation will be shown in Table 1.

##### Meson Lifetime

Using this distance for the lifetime in the equation given by Ninham [22], we obtain the  $\pi_0$  lifetime  $\geq 0.16 \times 10^{-16} \text{ s}$ . Noteworthy, as we mentioned earlier the predicted lifetime is stable for different nucleon-nucleon separations unlike binding energy (which increases with decreasing separations). This is a curious consequence of the density dependence of the plasmon wavevector. This is applicable only at the very high temperatures we predict (corresponding to a plasmon with energy high enough to create particles). The experimental textbook result [24] is around  $0.83 \times 10^{-16} \text{ s}$ . Our result is of the right order of magnitude. A “simple” QFT approximation [23] leads to an estimate for the lifetime around  $0.22 \times 10^{-16} \text{ s}$ . (A theoretically plausible improvement of the “simple” QFT result discussed by Weinberg [23] leads to  $0.52 \times 10^{-13} \text{ s}$  which is different by a factor 1000 from the experimental result). A better theoretical approximation, assuming among other

things the number of colors for the quarks, leads to an estimated QFT lifetime for the neutral pion of  $\sim 0.9 \times 10^{-16}$ s [23].

The decay of the neutral pion into two photons has its basis in the explicit breaking of the axial symmetry by quantum fluctuations of quark and gluon fields. The first four decay pathways [21] are: (1)  $\pi_0 \rightarrow \gamma\gamma$ , (2)  $\pi_0 \rightarrow \gamma + e^+ + e^-$ , (3)  $\pi_0 \rightarrow \gamma + \text{positronium}$ ; (4)  $\pi_0 \rightarrow e^+ + e^- + e^+ + e^-$ . Our theory, taken with the reactions  $e^+ + e^- \leftrightarrow \gamma$  and  $e^+ + e^- \rightarrow \text{positronium} \rightarrow \gamma$ , could account for the  $\pi_0$  particle being able to produce these four decay pathways. Precise measurements of the decay width of the  $\pi_0 \rightarrow \gamma\gamma$  process give an average of 7.80 eV. This gives a lifetime of  $0.834 \times 10^{-16}$ s [26,27]. This is in good agreement with previous theoretical results and with its estimated 1.5% accuracy offers a benchmark test for the most sophisticated theoretical estimates including the prediction  $0.804 \times 10^{-16}$ s by Kampf and Moussallam [28]. High accuracy calculations of the lifetime also include those discussed by Larin et al. [26] and by Bernstein and Holstein [29]. These authors [26,29] discuss how the axial, chiral, anomaly originating from quantum fluctuations of quark and gluon field, and exploiting the number of QCD quark colors, drives the  $\pi_0$  meson decay with a lifetime around  $0.849 \times 10^{-16}$ s.

### **Nuclear Binding Energy**

Furthermore, the Lifshitz-Yukawa binding energy at this separation receives -0.9 MeV from the  $n=0$  term and -3.6 MeV from the  $n>0$  term leading to a total binding energy from electromagnetic fluctuation interaction of 4.5 MeV. The binding energy increases with decreasing nucleon-nucleon separation in line with the fact that binding energies of nucleons are different in different nuclei [30-32], and also in line with the fact that local surroundings influence the local structure of the

nucleons [30-32]. The binding energy per nucleon varies in different atomic nuclei from 1.1 MeV for deuterium to 8.8 MeV for Nickel-62. Also, the structure of neutrons and protons within different nuclei depends on the local environment (for references see the work by Feldman [31]). (Note also in passing the experimental data on nucleon binding energies in Ref. [33]. In that (controversial) paper the authors infer that neutron-neutron, just as proton-proton interactions are repulsive, whereas the neutron-proton interaction is attractive.)

### Summary of Numerical Results

We summarize our numerical results in Table 1. The equivalent temperatures (note that:  $m_e c^2 / k \approx 5.9 \times 10^9 K$ ) are high enough to generate the electron-positron plasma. The effective surface area is taken to be  $A = \pi r^2$  with  $r = \text{proton radius} \sim 0.8$  fermi. Improved estimates would, for example, require an expansion of our planar estimate to consider a pair of perfectly conducting spheres in a high-density electron-positron plasma.

**Table 1:** The lifetime, meson mass and binding energy versus separation between a pair of neutrons (or protons). Recall the approximations in our model (Sec. 2.1), implying the nucleons to be replaced by conducting plates.

Separation	Lifetime	Meson Mass	Binding Energy	kT
1.0 fermi	$1.61 \times 10^{-17} s$	$491 m_e$	13.6 MeV	$193 m_e c^2$
1.5 fermi	$1.62 \times 10^{-17} s$	$267 m_e$	4.5 MeV	$128 m_e c^2$

2.0 fermi	$1.64 \times 10^{-17} \text{s}$	$173 m_e$	2.0 MeV	$97 m_e c^2$
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#### 4. Summary and Conclusion

We began this enquiry with the idea that if Feynman believed there ought to be a link between electromagnetic theory and nuclear forces, there might be something in it. It seems there is. From our semi-classical theory we have been able to predict better than order of magnitude estimates for the basic properties of the neutral pion, namely its decay length, mass, and lifetime. In the picture a high-density electron-positron plasma emerges quantitatively and naturally as a key player in nuclear interactions. A defect is the modelling of nucleon interactions by planar perfectly reflecting surfaces. There are two free length parameters, area and distance between the model "nucleons". But they are close to actual distance scales.

It would be more convincing if the theory also predicted the various decay modes for  $\pi_0$ , in terms of  $e^+e^-$  pairs and photons. Further, in such a theory the charged mesons,  $\pi^-/\pi^+$ , would emerge as an electron/positron bound to a plasmon.

One thing is clear. There is certainly enough energy available to account for nucleon interactions. And if the claim that our theory is not equivalent to the canonical theory, where has that energy gone? It is possible to push matters further by including magnetic susceptibilities in the formalism for interactions using a fully relativistic electron-positron plasma. There is a useful analytic framework available in the work of Daicic, Kowalenko, Frankel and co-workers [34-36]. In

connection with this we observe that Larin et al. [26] performed some of the most precise measurements of the lifetime of the  $\pi_0$  meson. Their weighted average final result for the  $\pi_0 \rightarrow \gamma\gamma$  decay width defines the new lifetime to be  $8.337 \times 10^{-17}$ s. Such surprisingly short lifetimes can in the QCD framework be obtained once axial anomaly is accounted for. The axial anomaly, which historically provided strong evidence in favor of the color-charge concept in QCD, seems to present us with state of the art knowledge about some of most fundamental aspects of nature—for example, by constraining the fundamental physics beyond the Standard Model and presenting opportunity to, e.g., measure the light quark mass ratio [26,27]. However, using a much simpler semi-classical theory we have found results that turn out to have exactly the right order of magnitude. This suggests an as yet unexplored link between our theory (expanded to magnetic anisotropic media) and one of the most profound theories in physical science.

There are wider implications: If the equivalence we seek can be firmed up, the consequences would be significant. The full QFT of interactions of DLP involves a nonlinear coupling constant integral equation for the polarisation operator. That awkward difficulty was resolved by replacing that by a linear integral, and the whole formalism collapsed to a semi classical theory.

The consequences of these mathematical simplifications have been a serious obstacle to progress in the biological and engineering sciences that depend on molecular forces in the disciplines of physical, colloid and surface chemistry [37-40]. The theories inconsistently treat electrostatic forces in a nonlinear theory and quantum fluctuation (dispersion) forces in a linear theory [37,41]. So central specific ion (Hofmeister) effects, and hydration effects are lost. The problem is being partially rectified [40]. But a proper fundamental theory requires the complete non linear theory to go further.

The same would be true for the theory of nuclear interactions. It should also be a non linear theory and not linear as it is now.

A partial version of this mss (BWN and Colin Pask, unpublished) was written in 1969. A brief version was published by two of us (BWN and MB) in 2003, but it omitted the meson lifetime. This version is more detailed and includes this important result. In the following 55 years what is new has been the application of Lifshitz theory to the foundations of physical chemistry. The literature is extensive. Classical theories ignore all important Hofmeister (specific ion effects). The problem can be traced to the same linearization approximation and rectified. The equivalence established between Lifshitz theory and  $\pi$  zero mesons implies that particle physics suffers the same difficulties.

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