Utilizing a Simple Numerical Model in Discrete Element Analysis to Simulate Flow Time and Number Tests of Asphalt Mixes

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Abstract: During the past decades, many numerical models have been used to predict responses of asphalt mixes under different types of loading. Some of these models were simple due to practicality but overestimated the response of asphalt mixes. On the other hand, sophisticated but effective numerical models have been developed to address the shortcomings of the simpler models, and were used mostly in finite element analysis (FEA). However, these models were complicated and not user friendly. Recently, the approach of the discrete element method (DEM) was adopted. Unlike traditional FEA, DEM can simulate crack propagation by allowing the separation of elements in the simulated models. Understanding these challenges, this study was initiated to investigate the utilization of a simple visco-elasto-plastic model that had been used successfully in predicting deformation in asphalt mixes using the DEM embedded in Particle Flow Code in Two Dimensions (PFC^{2D}) software simulations. Simulation results, when compared to flow time (F_T) and number (F_N) test results, showed that this model could simulate actual tests, thus predicting deformation of asphalt mixes using the DEM on a larger scale.

Keywords: Asphalt mixes, Numerical modeling, Discrete element method, Flow time test, Flow number test, Deformation.

الاستفادة من نموذج عددي بسيط لتحليل العناصر المتقطعة لمحاكاة اختبار تدفق الزمن و العدد لخلطات الأسفلت أحمد م. عبدم

الملخص: خلال العقود الماضية تم استخدام نماذج عددية كثيرة للتنبؤ برد الفعل لخليط الأسفلت لأحمال مختلفة. بعض هذه النماذج كان بسيطاً نتيجة للتطبيق العملي و لكن كانت مبالغة في تقدير رد الفعل لخليط الأسفلت. على الجانب الآخر تم تطوير نماذج عددية معقدة ولكن فعالة لإبراز أوجه القصور في النماذج البسيطة، وتم استخدام معظمها في تحليل العناصر المحددة. لكن هذه الطري و لكن حالت مبالغة عنه النماذج البسيطة، وتم استخدام معظمها في تحليل القرر تم تطوير نماذج عددية معقدة ولكن فعالة لإبراز أوجه القصور في النماذج البسيطة، وتم استخدام معظمها في تحليل العناصر المحددة. لكن هذه الطرق معقدة وليست ودية للمستخدم. اعتمد حديثاً نهج طريقة العنصر المتقطع. هذه الطريقة بخلاف طريقة العنصر المتقطع. هذه الطريقة بخلاف طريقة العناصر المحددة يمكنها محاكاة انتشار التصدع عن الطريق السماح بفصل العناصر في نموذج المحركاة. بعد فهم هذه التحديات، هذه الدراسة هدفت الى فحص استخدام نموذج فيسكو البلاستيكي المرن بنجاح في المحاكاة. بعد فهم هذه التحديات، هذه الدراسة هدفت الى فحص استخدام نموذج فيسكو البلاستيكي المرن بنجاح في المحاكاة. بعد فهم هذه التحديات، هذه الدراسة هدفت الى فحص استخدام نموذج فيسكو البلاستيكي المرن بنجاح في توقع التشوه في خلطات الأسفلت باستخدام طريقة العنصر المتقطع المرتبط ببرنامج المحاكاة. نتائج المحاكاة توقع التشوه في خلطات الأسفلت باستخدام طريقة العنصر المتقطع المرتبط ببرنامج المحاكة. نتائج المحاكاة قورنت بنتائج تدفق الزمن و العدد وأظهرت أن هذا الاختبار يستطيع محاكاة الاختبارات الحقيقية و بالتالي يمكن التنبؤ مورنت بنتائج تدفق الزمن و العدد وأظهرت أن هذا الاختبار يستطيع محاكاة الاختبارات الحقيقية و بالتالي يمكن التنبؤ بالتشوه في خلطات الأسفلت باستخدام طريقة العنصر المتقطع على نطاق واسع.

مفاتيح الكلمات: خلطات الأسفلت ، التمثيل العددي ، طريقة العنصر المتقطع ، اختبار تدفق الزمن، اختبار تدفق العدد و التشوم

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1. Introduction

Flexible asphalt pavements are being subjected to more diverse and harsher loading conditions than ever before. However, even after adjusting design factors to account for conditions, adopted these the design techniques, which were developed using simple models, fall short of predicting failures as efficiently as they were able to do in the past. Additionally, it is difficult to take all properties into consideration while designing flexible pavement. Asphalt mix is a viscoplastic and anisotropic material. It varies in terms of its temperature, loading time, and aggregate orientation dependency. Thus, designers have chosen to overlook those properties in favor of simplifying the design process.

Plastic deformation, known as rutting, is the major distress pavement designers encounter. In order to prevent this type of premature failure and any undesired effects due to permanent deformations, as well as to account for the complex material properties of asphalt mixes, a more realistic model is essential. Incorporating the development and adoption of such material models into design methodologies will result in an improvement of performance and а reduction maintenance.

Recent studies simulated asphalt mixes' responses to different types of loading conditions. By utilizing numerical models to describe a mesh of elements that simulated test setups, numerical solutions based on finite element analysis (FEA) were found. A study by Buttlar et al. (2003) suggested that most existing analysis models did not directly account for the continuous grading of properties in flexible pavements. Thev presented the application of a numerical model embedded in FEA software for asphalt pavement analysis. Masad et al. (2005) used an anisotropic non-associated flow rule based on the Drucker-Prager yield criterion. The model parameters were related to the experimental measurements of aggregates' characteristics and microstructure damage, which was measured using X-ray tomography and image analysis techniques.

Tashman *et al.* (2005) introduced a microstructure-based viscoplastic continuum model for predicting permanent deformation

of asphalt mixes. This model took into consideration strain rate and confining pressure dependency, dilation, aggregate friction, anisotropy, and damage, all of which have a huge impact on the permanent deformation of asphalt mixes at high developed model temperatures. The predictions were in good agreement with the experimental measurements. A disadvantage of FEA is that it will not allow the separation of elements during analysis. Thus, no formation of micro or macro cracks in the modeling of asphalt mixes under loading will occur.

2. The Discrete Element Method (DEM)

Recently, more attention has been directed to the DEM, which was introduced in 1971 to analyze problems with rock mechanics, ice formation and flow in streams, as well as earthquakes, impacts, and explosion damage to structures. The discrete element algorithm models a continuum as a system of distinct, interacting, and general-shaped particles subjected to laws of motion and deformation, which are bonded together. When the maximum stress in a contact bond is reached, the bond is broken and a separation of these particles, evidenced as cracks, can be observed (Cundall and Strack 1979). In the DEM, the complex constitutive behavior of a material is simulated by associating simple constitutive models with each particle contact. Shear and normal stiffness, static and sliding friction, and inter particle cohesion are three of the simpler contact models that can be employed (You 2003).

You and Buttlar (2006) argued that the DEM is a fundamental way of looking at the complex behavior and heterogeneity of asphalt mixes. They suggested that the DEM could be used to simulate asphalt mixes responses under different loading and temperature conditions. They represented the DEM approach as a research tool for modeling asphalt mix microstructure.

Abbas *et al.* (2007) utilized the DEM to develop a micromechanical model that accounted for viscoelastic behavior of asphalt mixes. Asphalt mix microstructure was captured using gray scale images of vertically cut sections of the compacted samples. Collop *et al.* (2006) investigated the use of the DEM by utilizing the Burger model to describe the contact bonds in their analysis under compression. They argued that the behavior of the mixture would be dominated by asphalt binder and complex aggregates interlock effects could be minimized. It was found that tested mixes dilated when the ratio of compressive to tensile contact stiffness increases as a function of loading time.

Recent studies (Yang et al. 2012; Liu et al. 2012; Cai et al. 2013) suggested that the DEM, via Particle Flow Code in Two Dimensions (PFC^{2D}) and Particle Flow Code in Three Dimensions (PFC^{3D}) software (Itasca International Inc., Minneapolis, Minnesota, USA), could be used to predict the rutting resistance of asphalt mixtures more conveniently, by simulating the different test setups of asphalt mixes. The simulation results showed that the discrete element simulation and laboratory test had good correlation, which verified the applicability of these adopted models.

When it was developed, the main function of the DEM was to act as a tool to perform research to help with understanding the behavior of granular materials. However, it has been used recently in biomechanics, petrochemical engineering, fluid mechanics, and different structural engineering applications in which particle models were used to simulate material behavior in real engineering problems that involve complicated deformation patterns (Shibata et al. 2003; Goda and Ebert 2005; Mas Ivars 2006; Mahmoud et al. 2010; Jinag et al. 2010; Lau et al. 2011; Krabbenhoft et al. 2012; Abu Abdo et al. 2012; Abraham et al. 2013; Nakamura et al. 2013).

3. Scope

The main purpose of this study was to supplement previous research conducted in the area of hot mixture asphalt (HMA) evaluation. This study investigated the utilization of a simple visco-elasto-plastic model that had been used successfully in predicting deformation in asphalt mixes (Abu Abdo 2012). The goal of simulating laboratory tests in DEM was to verify the ability of this simplified predictive model to determine the simplified predictive model and the deformation of asphalt mixes on a larger scale under different loading conditions.

4. Particle Flow Code in Two Dimensions (PFC^{2D}) Software

PFC^{2D} is a platform for conducting the complicated calculations of the DEM, where representative elements containing several hundred particles are tested numerically. Continuum methods are used to solve real problems that involve complicated deformation patterns (Itasca 2006).

In PFC^{2D}, particles' interactions are treated a dynamic process with states of equilibrium developing whenever the internal forces balance. Each particle is assigned contact bonds to all adjacent particles and will break when ultimate strength is reached. Contact forces and displacements of a loaded body of particles are found by mapping the movements of the individual particles, which is dependable on the physical properties of the discrete system. The calculations performed in PFC^{2D} alternate between the application of Newton's second law of particles and a forcedisplacement law of contacts. Newton's second law is used to determine the motion of each particle arising from the contact and body forces acting on it, while the forcedisplacement law is used to update the contact forces arising from the relative motion at each contact (Itasca 2006).

Most numerical modeling of asphalt mixes under axial compressive loading uses the Burger model to describe contact bonds between particles in PFC^{2D} simulations (Schwarz and Weeks 1977; Ye et al. 2009; Zelelew and Papagiannakis 2009). In this study, the sea ice model (Schwarz and Weeks 1977; Mellor 1981 and Abu Abdo 2012) was used for its better representation of the contact bonds between particles. Commonly, sea ice, which is similar to asphalt mixes, exhibits time-dependency, mainly delayed elastic recovery and creeping. Many experiments have shown that ice strength has a dynamic value depending non-linearly upon the strain rate. The strain-stress relation that describes the sea ice behavior under loading is described in Eqn. (1).

$$e_{T} = \frac{\sigma}{E} + c \left(\frac{d_{o}}{d}\right) \left(\frac{\sigma}{E}\right)^{s} \cdot \left[1 - e^{-(a_{T}.t)^{b}}\right] + e_{vo} \cdot t \cdot \left(\frac{\sigma}{\sigma_{o}}\right)^{n}$$
(1)

where,

 σ is the applied stress (kPa),

E is the mix stiffness (kPa),

c is a material constant,

b is a time exponent for delayed elastic strain,

 a_T is a material constant (s⁻¹),

d is an average grain diameter (mm),

 d_o is a grain diameter (mm),

- *s* is the stress exponent for grain-bound sliding,
- *n* is the degree of viscosity power law, and

 e_{vo} is the viscous strain rate (s⁻¹).

A study by Abu Abdo (2012) suggested that the sea ice model could be used effectively in predicting deformation in asphalt mixes under static and cyclic loading conditions. The model parameters were easily determined, and could be adapted to many numerical methods and used effectively in predicting the deformation of asphalt mixes.

5. Experimental Verification

5.1 Test Setups

5.1.1 Static Creep/Flow Time (F_T) Test

A flow time (F_T) test is a triaxial static compressive creep test at which a total straintime relationship for a specimen is measured in the laboratory under unconfined or confined conditions. A F_T test is used to determine the instantaneous elastic and plastic components, as well as the viscoelastic and viscoplastic components of the material's response as shown in Fig. 1. The relationship could be divided into three major zones: the primary zone (initial stage); the secondary zone (linear portion); and the tertiary flow zone (the portion in which the strain increases significantly) at which failure occurs.

A study by Witczak *et al.* (2002) suggested that the higher the slope of the linear portion of the graph (secondary zone) the higher the permanent deformation under loading would be. Furthermore, the F_T is defined as the postulated time when shear deformation, under constant volume, starts. In this study, the specimens were subjected to a static axial load of 207 kPa and were conducted without confinement at a testing temperature of 54.4 °C.

5.1.2 Flow Number (F_N) Test

Another approach of measuring permanent deformation of an asphalt mix is the flow number (F_N) test. It is a triaxial repeated load test conducted with several thousand repetitions of a cyclic load, while recording the cumulative permanent deformation as a function of the number of load cycles, as shown in Fig. 2, where the relation could be divided into three major zones similar to the F_T test.

The F_N is defined as the number of load repetitions at which shear deformation, under constant volume, starts (Witczak *et al.* 2002). Specimens were tested using a loading cycle of 1.0 second. The loading cycle consisted of a 0.1 second haversine load of 207 kPa followed by a 0.9 second rest at a testing temperature of 54.4 °C.

5.2 Tested Mixes Properties and Samples Preparation

Two Superpave mixes with different properties were used and are listed in Table 1. Samples were compacted using a Superpave Gyratory Compactor (SGC) to a height of 175mm with 9% air voids. These samples were then cored and sawed to obtain a sample measuring 100 mm in diameter and 150 mm in height. These samples had a 7% air void, as per the sample preparation procedure for the F_T and Number Tests (Witczak *et al.* 2002).

5.3 PFC^{2D} Simulations and Results

To conduct simulations using PFC^{2D} software, tested specimens were modeled. The model developed initially for this study was constructed of a wide range of particle (sphere) sizes, thus approximating the irregular matrix of the cross-section of an asphalt mix sample (Fig. 3). During this initial phase, the goal was to create a model that simulated actual physical samples. Unfortunately, the analysis of such a model did not converge and no solution was obtained. Then it was decided to

model asphalt mix samples as a homogeneous set of 5,000 particles arranged in rows where a solution was obtained (Fig. 4). One of the major benefits of using a homogeneous particle configuration is the significant decrease of time needed to analyze the model by excluding packing procedure. Then contact bond properties were identified and modeled using the sea ice model Eqn. (1). To utilize the sea ice model, the model parameters should be







Figure 2. Typical Flow Number Test Results for Asphalt Mixes (After Witczak et al. 2002).

determined. These parameters were obtained from a previous study (Abu Abdo 2012) (Table 2).

Actual results of the F_T and F_N tests were used to determine the model parameters.

Mixes 1 and 2 had the same model parameters except for the e_{vo} . It was suggested that the e_{vo}

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parameter was related to the asphalt binder grade (viscous part of the mix) and since the tested mixes had different binders (Table 1) it was expected that the e_{vo} would be different for both mixes.

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Ball	

Figure 3. Original Element Assembly (Flow Time Test Model).



Figure 4. Final Element Assembly (Flow Time and Number Test Models).

Similar to F_T and F_N test setups, a simulated specimen using PFC^{2D} software was loaded by

assigning forces (constant for the F_T test and cyclic for the F_N test) to the bottom particles to

simulate the lower loading plate in both tests setups (Fig. 5). The displacements of top particles were prohibited to simulate fixed top plate that acts as a support.

Laboratory test results for the F_T test (Fig. 6) showed that Mix 1 failed (tertiary flow zone) at a lower number of cycles ($t_f = 1025$ sec), while

Mix 2 did not fail. Thus, it could be concluded that Mix 2 would perform better than Mix 1 under the same loading conditions. This could be explained by the better mix properties of Mix 2 (*ie.* larger aggregates sizes and higher binder grade).

Parameter	Mix 1	Mix 2
Job Mix Design Class	19.0mm	25.4mm
Binder Grade	PG 58-34	PG 70-28
Asphalt Content	4.6%	5.0%
G _{mm}	2.801	2.458
%Passing Sieve Size		
25mm	100 %	98 %
19mm	96 %	96 %
12.5mm	79 %	82 %
9.5mm	70 %	73 %
4.75mm	46 %	55 %
2.36mm	28 %	36 %
1.18mm	18 %	24 %
0.6mm	12 %	13 %
0.3mm	9 %	9 %
0.15mm	5 %	6 %
0.07mm	3.5 %	4.4 %

Table 1. Mix properties as detailed by Abu Abdo (2012).

Table 2. Sea ice model parameters for asphalt mixes as detailed by Abu Abdo (2012).

Parameter	Mix 1	Mix 2
8	1.229378	1.229378
a_T	3.14x10 ⁻⁰⁴ s ⁻¹	3.14x10 ⁻⁰⁴ s ⁻¹
c (Static loading)	7.5	7.5
c (Dynamic loading)	2.0	2.0
b	0.34	0.34
п	3	3
e_{vo}	6.59x10 ⁻⁰⁶ s ⁻¹	1.78x10 ⁻⁰⁷ s ⁻¹

Then laboratory test results were compared to the simulated tests. As shown in Fig. 6, PFC^{2D} simulation slightly over-predicted the total strain of Mix 1 until the F_T was reached,

where the tested samples started to flow and shear deformation began (tertiary flow zone). On the other hand, PFC^{2D} simulation approximately matched the measured values of Mix 2.

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Figure 5. Flow Time (FT) and Number (FN) Test Setups.



Figure 6-a. Measured vs. Fitted Model Results of Total Strain Determined in Flow Time Test (Mix 1).



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Figure 6-b. Measured vs. Predicted Results of Total Strain Determined in Flow Number Test (Mix 2).



Figure 7. Measured vs. Predicted Results of Total Strain Determined in Flow Number Test (Mix 2).

To confirm the effectiveness of the sea ice model under different loading conditions, Mix 2 was tested using the F_N test (cyclic loading) setup. As shown in Fig. 7, PFC^{2D} simulation results over-predicted total strain at an earlier stage (primary zone) and then under-predicted the total strain at the secondary and tertiary flow zones. Further, it was observed that the total strain predicted by PFC2D reached an upper limit. The model parameters might need fine tuning to achieve better results. In addition, it was speculated that if PFC^{2D} model was constructed of a wide range sizes of randomly distributed particles (Fig. 3), simulation results would be more accurate since it represents an actual cross-section of a tested asphalt mix sample.

6. Conclusions

Nowadays different and harsher loading conditions exist for asphalt pavements. Therefore, the current design methods, which are based on simple models and assume that asphalt mixes have an elastic and isotropic nature, are not efficient. These methods cannot accurately predict failure as before. A more realistic and applicable approach is needed to account for the visco-plastic behavior and anisotropic nature of asphalt mixes. To prevent failures and undesired effects due to permanent deformations, many numerical models were developed to predict permanent deformation in asphalt mixes. Unfortunately, models include many these specific parameters, which makes their use impractical. This study described the use of a simple viscoelasto-plastic model that has been used successfully in determining Sea Ice distresses under different wind loads. The model was used in the DEM, embedded in the PFC^{2D} software. PFC^{2D} was used to simulate the F_T and F_N tests. Two Super pave mixes were used; results of the F_T test showed that Mix 2 would perform better than Mix 1 under the same loading conditions. Furthermore, a PFC^{2D} simulation of the F_T test results slightly overpredicted the total strain of Mix 1 and approximately matched the measured values of Mix 2.

To validate the utilization of the adopted model under different loading conditions, a simulation of Flow Number test was conducted using Mix 2. Results by PFC^{2D} simulations over-predicted the total strain at the early stage then under-predicted the total strain at the final stages. In addition, it was observed that total strain reached an upper limit, it is suggested that the model parameters might need fine adjusting to achieve better results.

Overall, results from this study indicated that the sea ice model could be utilized effectively in the DEM via PFC^{2D} software. Additionally, flexible pavement simulation in PFC^{2D} could be adapted to predict deformation on a larger scale and under different loading conditions.

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