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MIMO Channel Gain Mechanisms Relative to SISO Channel

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الكسب في قنوات التعدد نسبة الى القناة الاحادية: دراسة لكسب قنوات الاتصال اللاسلكية متعددة المداخل والمخارج مقارنة بوحيدة المدخل والمخرج

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الملخص: بعكس تقنيات الاتصال اللاسلكية التقليدية تتمتع تقنيات الاتصال متعددة الهوائيات بآليات كسب مختلفة تجعلها جذابة جدا لتوفير اتصال لا سلكي بمعدل تدفق عالي، هذه الورقة تقدم دراسة لهذه الآليات وتركز على الحالة التي تكون فيها نسبة الاشارة المستقبلة الى الضوضاء مرتفعة، حيث أن هذه الحالة تعتبر واعدة من ناحية توظيف تقنيات الاتصال متعددة الهوائيات، نقوم في هذه الورقة باعادة كتابة معادلة السعة لقنوات التعدد نسبة إلى كسب القناة الاحادية. بهذا الاجراء يصبح مفهوم كل من كسب التجمع الفراغي وكسب القدرة اكثر وضوحا. استنادا لهذا التحليل نقترح طريقة للتبديل بين الارسال باستخدام التجميع الفراغي أو الارسال باستخدام تعددية الارسال. ندعم العمل المعروض في هذه الورقة بنتائج عددية التي تظهر أن نسبة المشاركة لكل آلية من آليات الكسب في السعة الكلية لقنوات التعدد تعتمد بشكل كبير على معامل ريشان (فمكىز) للقناة وعلى نسبة الاشارة المستقبلة الى الضوضاء وكذلك على عدد الهوائيات عند المرسل والمستقبل. الخوارزمية المقترحة تعطى كسب يصل الى ٢ ديسبل في معدل الخطأ نسبة إلى استخدام التجمع الفراغي فقط كطريقة إرسال.

كلمات الدليل: قنوات التعدد، التجميع الفراغي، كسب القدرة، التبديل بين التجميع الفراغي وكسب القدرة.

Abstract: In contrast to the rest of wireless communication technologies, multiple-input multiple-output (MIMO) technology enjoys different gain mechanisms that make it very attractive for reliable high data rate wireless communications. This paper presents a study on these gain mechanisms with particular emphasis on the case of high average received signal to noise ratio (SNR) where the MIMO system deployment is most promising. We write the MIMO channel capacity in terms of gains relative to a single-input single-output (SISO) wireless channel. Doing so, spatial multiplexing gain and power gain of MIMO wireless channels become more insightful. Based on this analysis a switching scheme between spatial multiplexing and transmit diversity is proposed. We support our discussion with numerical results which show that under a high data rate spatial multiplexing scheme the contribution of each gain mechanism to the total channel capacity depends on the channel Ricean factor, the average received SNR, and the MIMO system size. The proposed switching scheme gives about 2 dB gain in bit error rate performance relative to the spatial multiplexing mode.

Keywords: MIMO channel, Gain mechanisms, Channel capacity, Spatial multiplexing gain, Diversity gain, Switching scheme

1. Introduction

Multiple-input multiple-output (MIMO) wireless technology is one of the promising technologies to cope with the increasing demands for high data rate wireless communications in an efficient way (Foschini and Gans, 1998). The attractiveness of this technology is due to the fact that there are two gain mechanisms contributing to the high performance achieved

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by MIMO systems. These mechanisms include spatial multiplexing gain and power gain (Telater, 1999 and Anderson, 2000). The number of spatial parallel channels created within the same allocated radio spectrum represents the spatial multiplexing gain. Mathematically, this corresponds to the number of nonzero eigenvalues of the normalized channel correlation matrix. On the other hand, the power increase in the average received SNR relative to zero dB represents the power gain obtained due to MIMO technology. At maximum, this gain can be up to the Frobenius norm of the normalized channel matrix. It is widely known that the gain obtained from these two gain mechanisms depends on the employed signaling scheme and the SNR (Zheng and Tse, 2003). Maximizing the gain due to one mechanism consequently reduces the gain due to the other one. A fundamental tradeoff curve between the spatial multiplexing gain and diversity gain was developed in (Zheng and Tse, 2003).

Due to the fact that the contribution of each gain mechanism change with MIMO channel realization, several switching schemes between different MIMO operation modes were proposed (Heath and Paulraj, 2005; Forenza and McKay, 2007; McKay *et al.* 2007 and Choi and Alamouti, 2008). In (Heath and Paulraj, 2005) a switching scheme between spatial multiplexing and transmit diversity is proposed. Switching between statistical beamforming, double space-time transmit diversity, and spatial multiplexing based on channel statistics is proposed in (Forenza and KcKay, 2007). In (McKay *et al.* 2007) a switching scheme between beamforming and multiplexing based on closed form bit error rate (BER) approximations is proposed. In (Choi and Alamouti, 2008) a physical abstraction and switching algorithm is proposed. A comprehensive review of recently proposed framework for adaptive MIMO architecture is presented in (Chae *et al.* 2010). In (Ball *et al.* 2009) performance comparison between open loop and closed loop MIMO schemes for broadband radio access is presented.

The meaning of MIMO channel gains can be made more meaningful by relating the MIMO gain mechanisms to a single-input single-output (SISO) wireless channel. In this paper, we use the MIMO channel capacity to relate the two MIMO gain mechanisms to a SISO wireless channel of the same propagation environment. In this way, the contribution of each gain mechanism to the MIMO channel capacity becomes clear and the meaning of the gain mechanisms is more insightful. Furthermore, based on this analysis we propose a new switching scheme between spatial multiplexing and transmit diversity. The proposed scheme is very simple to implement and it requires only one bit information for feedback from the receiver to the transmitter.

The rest of this paper is organized as follows; Section II describes the system model and the considered signaling scheme. Section III relates the MIMO channel gain mechanisms to SISO channel using the MIMO channel capacity formula. A simple switching scheme is proposed in Section IV. Numerical results and insightful discussions are presented in Section V. The conclusions of this work are drawn in Section VI.

2. System Model and Signaling Scheme

Consider a narrow band MIMO wireless communication system employing N_t transmit antennas and N_r receive antennas. With high data rate spatial multiplexing scheme, where different transmit antennas are fed with different streams of data that have zero mean circularly symmetric complex Gaussian (ZMCSCG) distribution, the error free spectral efficiency of this MIMO system is given by, (Foschini and Gans, 1998),

$$c = \max_{Tr(\mathbf{Q})=N_t} \log_2 \det\left(\mathbf{I}_{N_r} + \frac{1}{\sigma_n^2}\mathbf{HQH}^H\right) \tag{1}$$

where $\mathbf{H}^{N_r \times N_t}$ is the narrow band normalized channel matrix, $\mathbf{Q} = \mathbf{E}[\mathbf{x}^H\mathbf{x}]$ is the covariance matrix of the transmitted signal satisfying the power constrain $Tr(\mathbf{Q}) = N_t$, $Tr(\cdot)$ denotes the trace of matrix, σ_n^2 is the noise power at each receive antenna and $(\cdot)^H$ represents Hermitian transposition. In (1), it is assumed that $N_r \geq N_t$. When there is no channel knowledge in the transmitter side the optimal power allocation strategy is to distribute the total available power between the transmit antennas equally. Mathematically this implies that the transmitted signal covariance matrix is a diagonal matrix with entries of $\frac{\sigma_s^2}{N_t}$ where σ_s^2 is the total available power at the transmitter side. Therefore, the channel capacity in (1) can be rewritten as,

$$c = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{R}\right) \tag{2}$$

where $R = HH^H$ is the channel correlation matrix and is the average received SNR. This power allocation strategy results in loss of the array gain since the transmitter array gain is not realizable with this power allocation strategy (Anderson, 2000). Writing the channel capacity in terms of the eigenvalues of the channel correlation matrix reveals valuable information about the different MIMO channel gain mechanisms. Decomposing the channel correlation matrix to its eigenvalues, the channel capacity can be rewritten as,

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$$c=\Sigma_{i=1}^{r(\mathbf{R})}\log_2\left(1+\frac{\rho}{N_t}\lambda_i(\mathbf{R})\right) \quad (3)$$

where $\lambda_i(\mathbf{R})$ and $r(\mathbf{R})$ are the i-th eigenvalue and the rank of the channel correlation matrix, respectively.

3. MIMO Gain Mechanisms Relative to SISO Channel

The channel capacity in (3) reveals the following useful information about the MIMO system. There are $r(\mathbf{R})$ spatial parallel channels, each has an average SNR ρ/N_t and power gain of $\lambda_i(\mathbf{R})$. These are valuable information for MIMO system performance prediction. However, the quality of these parallel channels is not clear. The quality of parallel channels in line of sight (LOS) scenario is different than parallel channels in non-LOS (NLOS) scenario. The above information can be made more useful by making the discussion relative to a single antenna transmission system. This simply can be done by multiplying and dividing each eigenvalue of the channel correlation matrix by the power gain of a single antenna system λ_s. The SISO channel power gain λ_s is the power gain of one of the sub channels from the channel matrix. Without loss of generality we select $\lambda_s = h_{1,1}h_{1,1}^H$, where $h_{1,1}$ is the subchannel between the first transmit antenna to the first receive antenna. Doing so the MIMO channel capacity with MIMO gains relative to the single antenna system can be obtained as

$$c=\Sigma_{i=1}^{r(\mathbf{R})}\log_2\left(1+\frac{\rho}{N_t}\lambda_s\frac{\lambda_i(\mathbf{R})}{\lambda_s}\right) \quad (4)$$

It is well known that the promising advantages of MIMO wireless communications systems strongly depend on the average received SNR. In a low average received SNR scenario, the MIMO system is reduced to spatial diversity system and only the power gain mechanism is realized. On the other hand, at a high average received SNR both the spatial multiplexing gain and power gain mechanisms can be realized. Under high SNR scenario and full rank channel correlation matrix assumptions the channel capacity in (4) can be decomposed as

$$\begin{aligned} c &\approx \Sigma_{i=1}^{r(\mathbf{R})}\log_2\left(\frac{\rho}{N_t}\lambda_s\frac{\lambda_i(\mathbf{R})}{\lambda_s}\right) \\ &= r\log_2\left(\frac{\rho}{N_t}\lambda_s\right)+\Sigma_{i=1}^{r}\log_2\left(\frac{\lambda_i(\mathbf{R})}{\lambda_s}\right) \end{aligned} \quad (5)$$

where $r = min\ (N_r, N_t)$ is the rank of the channel correlation matrix.

Using this formulation, the information we get from the channel capacity becomes more insightful. The first term in (5) represents the obtained spatial multiplexing gain (SMG) which means that utilizing MIMO technology will offer r spatial parallel channels. The quality of these channels depends on the propagation environment. Generally, the power gain of a SISO channel in presence of LOS component is higher than that when there is no LOS component. It means that under LOS condition we expect to have better spatial parallel channels compared to the NLOS case. According to (5), for full rank channel matrices, one expects to have a higher channel capacity due to the spatial multiplexing gain in LOS scenario compared to the NLOS scenario.

The second term in the channel capacity formula (5) informs us about the power gain (PG) in each spatial parallel channel relative to the single antenna system. This makes the power gain term more informative. The power gain in each channel includes both the array gain, in our case only receiver array gain, and the diversity gain. However, it is very difficult to decompose the two power gains into individual quantities. Under LOS conditions, little power gain is achieved because the power of the SISO channel in general will be high compared to the NLOS case. Therefore, one expects to have low contribution in the channel capacity due to the power gain under LOS conditions, while higher contribution is expected due to the power gain mechanism under NLOS condition.

4. Switching Scheme

From the above discussion it can be noticed that when the contribution of the SMG mechanism to the channel capacity is more than that of the PG mechanism, then it can be deduced that the channel supports the spatial multiplexing mode more than the transmit diversity mode. On the other hand when the channel characteristics change and the channel capacity due to PG becomes higher than the channel capacity due to spatial multiplexing, it means that the channel supports transmit diversity mode more than spatial multiplexing mode.

Figure 1 shows a block diagram of the proposed method. At the transmitter there are two possible modes of operation, spatial multiplexing mode or transmit diversity mode. In spatial multiplexing mode the input data stream is divided into independent streams to be sent simultaneously on different antenna elements. The space-time receiver decodes the received signal to provide an estimate of the transmitted data streams. In diversity mode, the same input data stream is sent over multiple antennas at the same time. The space-time receiver in this case simplifies to a maximum ratio combining that decodes the received

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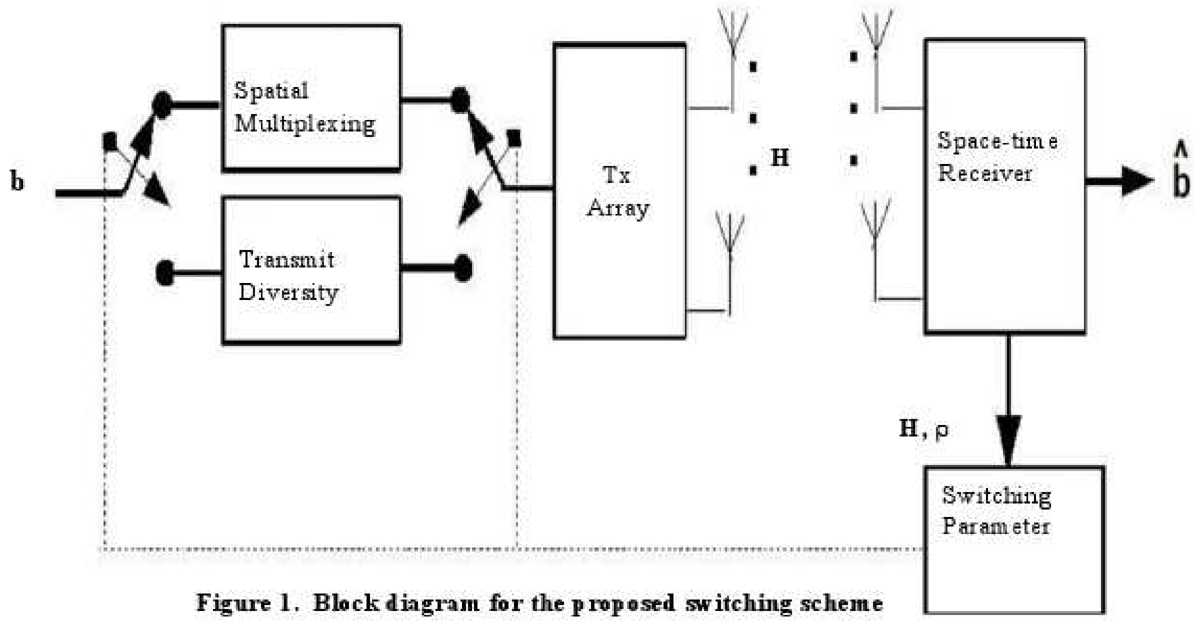

Figure 1. Block diagram for the proposed switching scheme

signal to provide an estimate of the transmitted data stream. In any case, the receiver also delivers information about the channel state information and the SNR. The channel matrix and the SNR are used according to (5) to evaluate the capacity due to the PG and SMG terms. The receiver decides the best mode of operation in order to maximize the channel capacity and sends back a one bit command to the transmitter in order either to switch or not from the current mode of operation. The two important measures that can be used to quantify the performance of the system are the BER and the data rate. Operating in SM mode increases the data rate, while operating in diversity mode the BER performance is enhanced. This will be investigated more in the numerical simulations section.

It should be noticed that the proposed scheme works on instantaneous basis, meaning that the instantaneous channel variations will affect the switching behavior. The proposed algorithm requires full channel state information. It means that a fast channel estimation method is required in order to work on instantaneous channel state measurements especially for large MIMO systems with large number of terminal antennas. A variation of the proposed technique is to send mode switching commands based on average measurements taken over a certain period of time related to the coherence time of the channel. This will reduce the number of times the switching is performed and reduces the likelihood of erroneous switching due to imperfect channel estimates.

5. Numerical Results

In this section we demonstrate the usefulness of relating MIMO channel gain mechanisms to single antenna systems numerical results. Two types of MIMO channels are considered, uncorrelated Rayleigh channel and spatially correlated Ricean channel with different Ricean factor. In the following presented results the channel matrices are generated according to

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{los} + \sqrt{\frac{1}{K+1}}\mathbf{H}_{nlos} \tag{6}$$

where K is the Ricean factor, and H_{los} and H_{nlos} are LOS and NLOS components, respectively. The Ricean factor K represents the percentage of the power of the LOS component relative to the power of the NLOS components. It is an indication for the channel correlation state. The case $K = 0$ represents low spatially correlated scenario and the case $K = \infty$ represents highly correlated scenario. The value of the Ricean factor has significant impact on the MIMO channel properties. One indication of MIMO channel properties is the difference between the minimum and the maximum eigenvalue of the channel correlation matrix in dB scale which is known as the channel condition number. Large condition number means bad MIMO channel.

Figure 2 shows the channel condition number as a function of the channel Ricean factor for MIMO systems with sizes ranging from 2×2 to 10×10. The effect of the channel Ricean factor can be clearly seen on the channel condition number. In case of 2×2 MIMO system the channel condition number change from -7.5 dB at $K = 0$ to about -20 dB at $K = 5$. With increasing the MIMO system size the impact of the Ricean factor on the channel condition number becomes significant. For instance with 6×6 MIMO system the change in the Ricean factor from $K = 0$ to $K = 5$ results in about 15 dB difference in the channel condition number.

Figure 3 shows the ergodic channel capacity of different MIMO systems with different Ricean factor at

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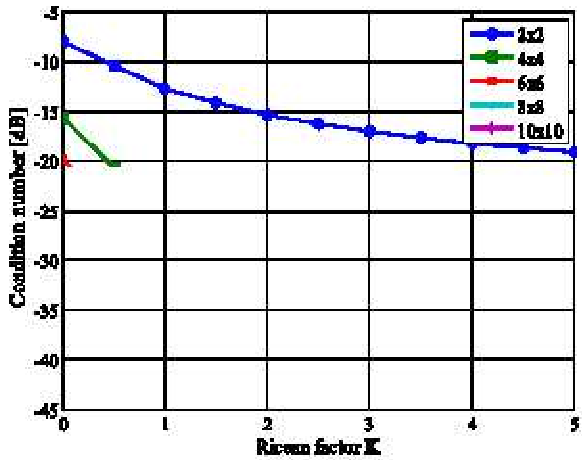

Figure 2. Channel matrix condition numbver and Rician factor for different MIMO systems

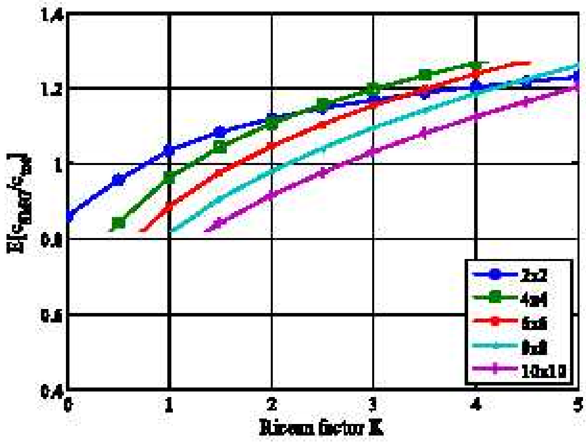

Figure 4. Average relative contribution of SMG on the total channel capacity of different MIMO systems at 20 dB SNR

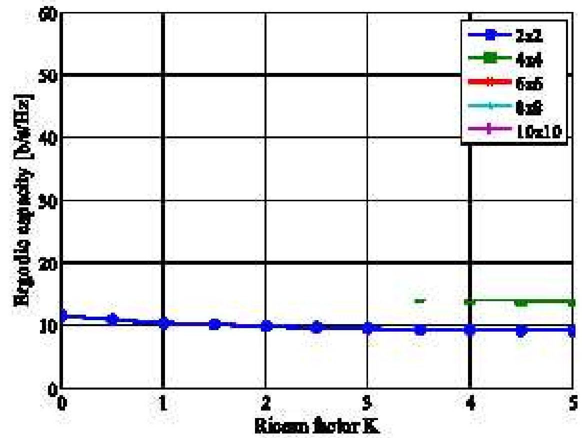

Figure 3. Ergodic capacity of different MIMO systems and channel Recian factor at 20 dB SNR

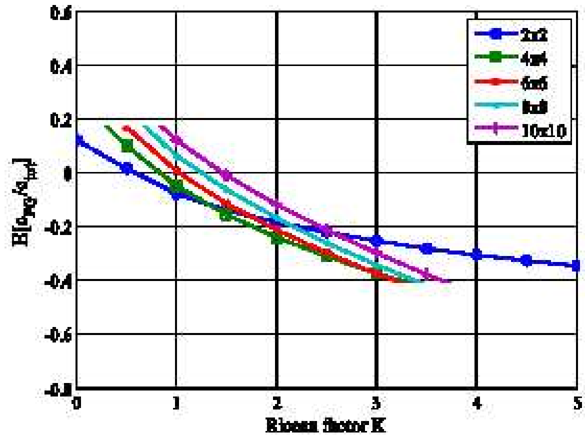

Figure 5. Average relative contribition of PG on the total channel capacity of different MIMO systems at 20 dB SNR

20 dB SNR. The impact of the channel Ricean factor on the ergodic capacity can be clearly seen. As the channel Ricean factor increases the achievable channel capacity decreases. The reduction in the channel capacity is more significant in large MIMO systems than in small MIMO systems. For example, changing the channel Ricean factor from 0 to 5 results in about 3 b/s/Hz loss in the channel capacity of 2×2 MIMO system. On the other hand, with 6×6 MIMO system the same change in the Rician factor results in about 15 b/s/Hz reduction in channel capacity. This is because large MIMO systems are more sensitive to changes in Ricean factor than small MIMO systems.

The average relative contribution of the spatial multiplexing gain (SMG) on the total channel capacity for different MIMO systems at 20 dB SNR and under different channel Ricean conditions is shown in Fig. 4. It can be observed that the contribution of the spatial multiplexing mechanism on the total channel capacity depends largely on the channel Ricean factor. With small channel Ricean factor the channel capacity obtained through SMG is relatively small compared to the case with large channel Ricean factor. For instance with 6×6 MIMO system about 60% of the channel capacity is obtained due to spatial multiplexing mechanism at $K = 0$. This percentage of capacity due to SMG raises to about 130% with $K = 5$. The explanation of this result is that as the Ricean factor increases the quality of the spatial parallel channels becomes better which make them more capable of carrying higher data rates. We can also note that the contribution of spatial multiplexing mechanism on the total channel capacity also depends on the MIMO system size. Figure 5 shows the average relative contribution of the power gain (PG) mechanism on the total channel capacity for different MIMO systems at 20 dB SNR and under different channel Ricean conditions. It can be seen that the contribution of the power gain mechanism on the total channel capacity is a decreasing function of the channel Ricean factor regardless to the MIMO system size. This is due to the fact that as the Ricean factor increases the power of the SISO channel becomes higher and the channel condition number becomes smaller which reduces the contribution of the power gain mechanism to the total channel capacity.

The contribution of each gain mechanism on the total channel capacity depends also on the operating SNR. Figure 6 shows the average relative contribution of the spatial multiplexing gain on the total channel capacity of 6×6 MIMO system at different channel Ricean factors. As it can be seen that at low SNR the contribution of SMG is small and it increases with the increase of the channel Ricean factor. For instance at 10 dB SNR the contribution of the SMG mechanism is about 0 % at $K = 0$ and about 50 % at $K = 0$. At high SNR most of the channel capacity is due to SMG mechanism. This is expected because of the SNR parameter is contributing to the the capacity due to the SMG mechanism (5).

Figure 7 shows the average relative contribution of

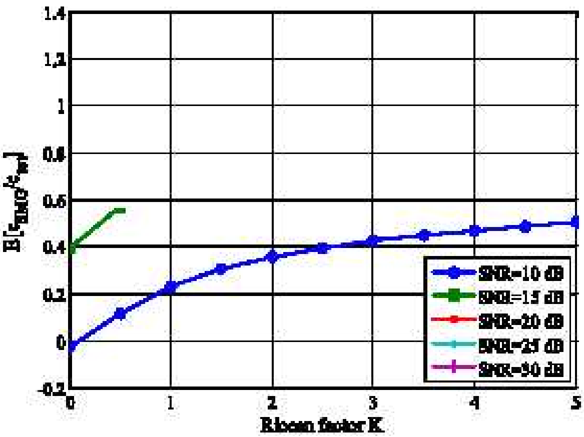

Figure 6. Average relative contribution of SMG on the total channel capacity of 6 x 6 MIMO system and at different SNR

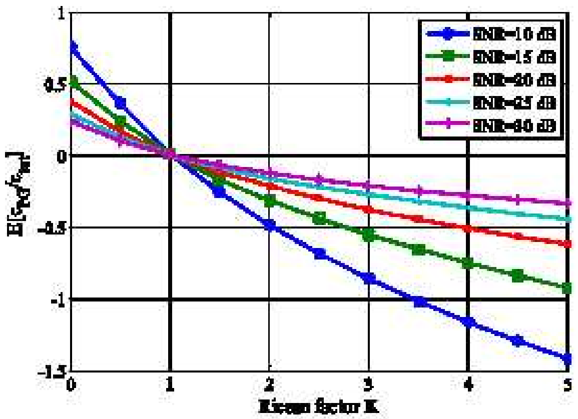

Figure 7. Average relative contribution of PG on the total channel capacity for 6 x 6 MIMO system at different SNR

the power gain mechanism on the total channel capac-

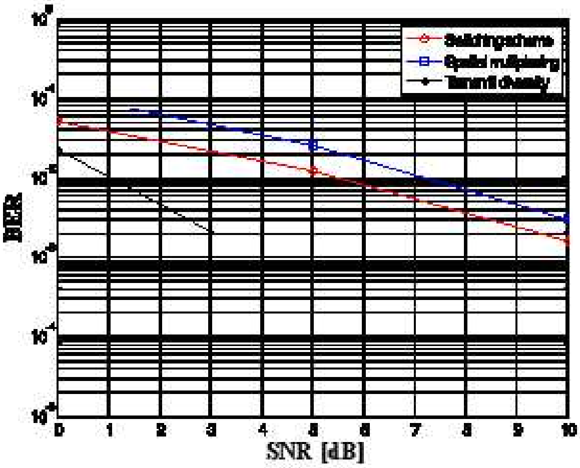

Figure 8. BER performance of different MIMO operation modes at $K = 5$

ity of 6×6 MIMO system at different channel Ricean factors and SNR values. It can be seen that as the SNR increases the contribution of the power gain on the channel capacity decreases. With low channel Ricean factor and low SNR the contribution of power gain on the channel capacity is significant. When both the SNR and the channel Ricean factor increases the contribution of the power gain starts to decrease. At channel Rician factor above $K = 1$ the contribution of the power gain mechanism becomes negative.

The performance of the proposed switching scheme was investigated via simulation. A 2×2 MIMO system which supports two operating modes, spatial multiplexing mode and transmit diversity mode, is assumed. The transmit diversity mode is Alamouti scheme (reid Harrison *et al.* 2007). The system has accurate channel state information and it uses this information to decide which operation modes to use. The bit error rate performance of the considered system is shown in Fig. 8 at channel $K = 0$. As it is shown the best error rate performance is achieved when the system operates in transmit diversity mode and the worst error rate is when the system operates in spatial multiplexing mode. This is due to the fact that transmit diversity can realize both transmit diversity gain and receive diversity gain while spatial multiplexing can realize receive diversity only. With switching scheme the BER rate performance of the system improves significantly compared to the spatial multiplexing mode. At 5 dB SNR the gain due to switching scheme is about 2 dB. The gain obtained when using the switching algorithm depends on the channel Ricean factor. Figure 9 shows the BER performance for the same MIMO system at K = 5. As it can be seen the gain due to the switching scheme is reduced specially at high SNR values. Since a channel with high Ricean factor and high SNR, the total channel capacity is dominated by the capacity due to spatial multiplexing gain and therefore, the proposed switching scheme does not provide significant

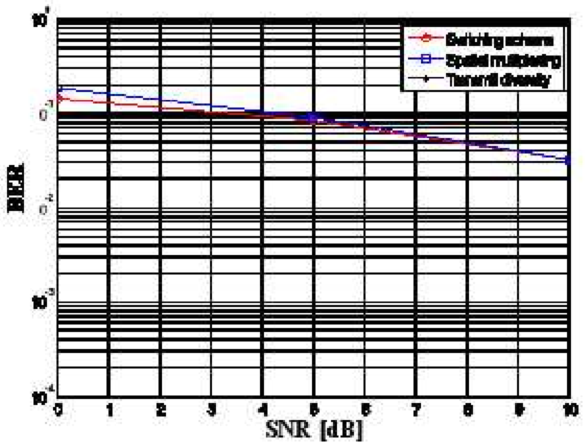

Figure 9. BER performance of different MIMO operation modes at K = 5

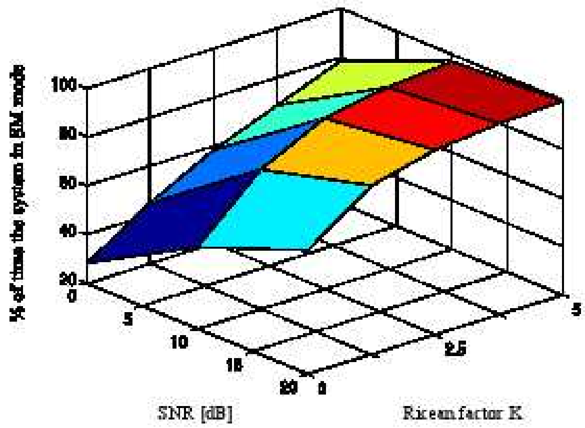

Figure 10. Percentage of time the MIMO system operates in spatial multiplexing mode

BER improvement in such situation.

Figure 10 shows the percentage of time the 2×2 MIMO system operates in spatial multiplexing mode for different channel Ricean factor and SNR. At low SNR and K factor, about 30% of the time the MIMO system operates in spatial multiplexing mode and 70% of the time in transmit diversity mode. As either the SNR increases or the Ricean factor increases the system will tend to operate in spatial multiplexing mode. At high Ricean factor channel condition the system operates most of the time in spatial multiplexing mode regardless to the SNR. Similarly, in high SNR scenario the system tends to operate in spatial multiplexing mode. This result is in line with the observation from the MIMO gain mechanisms above.

Conclusions

In this paper we have presented discussions of MIMO channel gain mechanisms and proposed a simple switching scheme between two MIMO system operating modes, namely, spatial multiplexing and transmit diversity. Our discussion of MIMO channel gains is based on relating the MIMO channel capacity to a single antenna system. We have shown that the meanings of MIMO gain mechanisms can be made more insightful by relating the spatial multiplexing gain and the power gain to a single antenna system. With numerical results we have shown that the contribution of each MIMO gain mechanism depends on the MIMO system size and the channel Ricean factor. The proposed switching scheme shows significant improvement in the bit error rate performance of MIMO system with negligible feedback information.

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