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Adaptive Filter based on Absolute Average Error Adaptive Algorithm for Modeling System

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ABSTRACT

Adaptive identification of the bandpass finite impulse response (FIR) filtering system is proposed through this paper using variable step-size least mean square (VSS-LMS) algorithm called absolute average error-based adjusted step-size LMS as an adapted algorithm. This algorithm used to design an adaptive FIR filter by calculating the absolute averaged value for the recently assessed error with the previous one. Then, the step size has been attuned accordingly with consideration of the slick transition of the step size from bigger to smaller to score an achievement through high convergence rate and low steady-state misadjustment over the other algorithms used for the same purpose. The simulation results through the computer demonstrate remarkable performance compared to the traditional algorithm of LMS and another VSS-LMS algorithm (normalized LMS) which used in this paper for the designed filter. The powerful of the filter has been served in the identification system, bandpass filter has been chosen to be identified in the proposed adaptive system identification. It reports conceivable enhancements in the modeling system concerning the time of convergence, which is well-defined as a fast and steady-state adjustment defined with a low level. The designed filter identified the indefinite system with less than 10 samples; meanwhile, other algorithms were taking more than 20 samples for identification. Alongside the fine behavior of preserving the tradeoff between miss adjustment and the capability of tracking, the fewer calculations and computations regarding the algorithm requirement make the applied real-time striking.

Index Terms: Adaptive Filtering System, Finite Impulse Response design, System Identification, Variable Step-size least mean square algorithm, Absolute Average Error-Adjusted Step Size

1. INTRODUCTION

The main concern of system modeling is a system estimation basis on data observation. This includes the requirement of the model structure, parameters prediction of the unknown plant, and proof of the resulting plant for input-output systems [1]. The main goal of system discovery is to provide a suitable mathematical model or equation for the unknown system that is considered; then, the model is constructed mathematically as a reference based for the next identification design [2]. For multiparameter systems such as filters, a typical optimization problem occurred, the filter parameters must be chosen to reduce the difference between measured and predicted filters to the minimum state. The difficulty level increased by increasing those parameters, especially with different scales, and the noise and incomplete measurements. Furthermore, the reference filter that has been selected has a major effect on the accuracy and strength of the procedure of identification [3]. The progression of computer algorithms causes an advancement in modeling for a variety of systems. Different intelligent methods are proposed for highly accurate estimation for both non-linear and linear systems [4]. In this work, an adapted filter was designed with different adaptive algorithms and engaged in the modeling system to identify a bandpass filter (BPF) with the minimum difference

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between filter coefficients and output signal. An adaptive filter (AF) basically is a digital filter which has a changeable transfer function and adjustable weights (taps), the change is implemented based on the designed filter [5]. The filter weights are adjusted using different techniques being used for implementations in several applications such as system identification, noise cancellation, acoustic echo cancellation, channel estimation, and adaptive line enhancement, system identification is considered as the most crucial application of adapted filters as this work presents. In order of adaptation process, different algorithms are employed for the weights adjustable process useful and effective depending on their concentrated arithmetic structure, leading to efficiency improvement of the performance [6]. This research paper deals with the execution of the LMS, normalized LMS (NLMS), and absolute average error-based adjusted step-size LMS (AAE-ASSLMS) algorithms. The algorithm of LMS is considered an exceptional algorithm of gradient search. The algorithm is often used for many applications on AFs assignable to its simplicity in implementation and the small number of calculation requirements, leading to its usage in countless applications [7]. In most adaptive system applications, fast convergence and low misadjustment are a demand. Although, the algorithm comes down with decelerate convergence time which is conversely followed by the step size. Yet, the selection of large adaptation step size yields into fast convergence but this selection outcomes downslope the steady-state attitude which is directly involved in the misadjustment, this conciliation leads to weakening the LMS functionality. As such, several researchers continually looked for alternative approaches to enhance its performance [8], [9]. From the common approaches, regarding the time-varying step size for updating the weight is the NLMS algorithm, furthermore, the AAE-ASSLMS algorithm is adopted in the filter design, and a comparison is made between the adaptive algorithms to highlight the performance improvement in means of convergence time and the misadjustment in the steady state.

2. LITERATURE REVIEW

The systems with AFs are used for various applications, because of their stability, effectively support many applications, the capableness of implementation, and linear phase response production. Several researchers proposed either new or modified adapted algorithms which are employed in various filter designs then those filters had been served in different systems through many fields as a means of achieving performance enhancement of the designed filter and the system that filter served in it [10]. Proposed a system to enhance an algorithm for comb adaptive filtering to manage the response equalization for the multipath communication channel, the system provides a high-speed tuning in the receiver side where the multispeed adaptive system is, and the processing of the computational costs in real time is obviously reduced, beside the algorithm analysis. As a result, the developed algorithm provided rate advancement and computational reduction complexity compared with LMS. A boosted AF is proposed by Kari *et al.* [11] using many adapted filtering algorithms those are RLS, LMS, and piecewise linear regression, the proposed algorithm mixture improved the performance opposite to the corresponding adaptive filtering approaches under the same used statistical conventions by establishing gains to mean square error (MSE). Through the years, lots of approaches had been proposed regarding varying the step size [12], presented a combination of adapted filters to enhance its performance, combinations of two algorithms (RLS and LMS) are considered with performance analysis as attested by the time-varying solutions. That convex combination between those algorithms gave the opportunity of achieving performance close to the optimal EMSE and MSD with lower computational complexity. Since LMS has a fixed step size, Si-Min *et al.* [13] proposed a system that enhanced the algorithm by investigating the skewness prediction distribution of LMS parameters as a third-order statistical feature in the purpose of vibration signal filtration system, the outcome shows improvements in error measurement angles and attitude accuracy. Meanwhile, Silva *et al.* [14] adopted (LMS) for adapting a system that assesses the vibration measurement of the drilling tool, the outcome shows measurement accuracy enhancement in the term of steerable drilling tools. The output demonstrated the feasibility of skewness growing limitless despite the existence of adaptive weights which present convergence in average and the mean square attitudes.

Another system built by Yuan and Songtao [15] depending on the non-linear relational band between step size and error, the new variable step-size least mean square (VSS-LMS) proposed and analyzes with different parameters. This brand new algorithm is overcome the step-size adjusting shortage of SVSLMS which is applied on the Sigmoid function, computer simulations and theoretical analysis show advancement with regard to convergence. Another research [16] had the same concern in a try to break the struggle between the time of convergence with the misadjustment of steady state by presenting a control system with a designed filter using an extended varying step-size algorithm, the step-size formula is modeled to improve the algorithm's ability of noise interference resistance. Else more, a VSS-LMS was proposed [17] using squared error autocorrelation criterion to administrate step-size value variation, a smoothing function employed for such purpose. This approach employs two quantities of step size, a large one for fast convergent in transient state and a small one for low (MSD) level in steady state. The proposal's performances have shown through simulation outcomes illustrated the good performance of the design over other algorithms under the same conditional test. To optimize the issue of both convergence and low misadjustment problems, Rusu *et al.* [18] produced a varying step size-based algorithm to turn this misalignment to its minimum by optimizing both LMS besides NLMS control time-varying parameters. The theoretical outcomes have been confirmed by the output simulation and show well features of that algorithm.

In the frame of the issue, Huang *et al.* [19] produced a novel diffusion (DRVSS-LMS) algorithm which is insensitive to deal with impulsive noise in a network, the proposal is about assigning VSS aiming to improve estimation performance, the algorithm recorded an achievement through high convergence rate and low steady-state misadjustment better than existing diffusion LMS algorithms with impulsive noise. Reviews of seventeen different (VSS-LMS) algorithms introduced by Bismor *et al.* [20], their performances are compared within three various applications systems: System identification, noise removal, and line enhancement, it also presents a modification suggestion for step size that is a central parameter for several LMS algorithms. Surfing through the previous proposing and modifying algorithms to achieve fast convergence with low misadjustment then get the benefit of such improvement of filter performance to boost the efficiency of a specific system, a VSS algorithm is employed in this study to design finite impulse response (FIR) filter then employed in identification system to identify a BPF. A new adjusted VSS algorithm proposed by Jamel and Mohamed [21] called AAE-ASSLMS and employed it into ANC system to enhance the quality of noise cancellation regarding input speech signal, the same algorithm employed in this paper for AF design which is served in modeling system that used to identify a BPF with optimal fast conversion rate, low level of misadjustment, and minimum samples for identification.

3. ADAPTIVE MODELING SYSTEM

The main purpose of the modeling system is to identify unknown systems or plants, the identifier system designed in this paper is to identify a BPF. The system employs an FIR filter serving the purpose efficiently, giving the best prediction of the unidentified system by providing a linear model. A general block diagram of system modeling is presented in Fig. 1 [1].

Parallel connection takes place gathering both the unidentified system (to recognized) with the designed FIR filter. A general input-output relationship is described by $x(n)$ and $d(n)$, respectively, to be an input to the indefinite system and its output. Besides the mysterious system, $x(n)$ feeds as input to the meant filter that is by its linear model provides the best description to the relationship of input with output to achieve the modeling by producing the nearest matching with the indefinite system output $d(n)$ through its output represented by the signal $y(n)$. On the same end, an error has been computed $e_1(n)$ showing the accuracy of identification by comparing the outputs of that filter in interest and the undefined system [1], [2]. In this work a BPF used as an unknown system to be identified by the required adaptive filter of the system model.

As believed by the major role of the filter adaptation in the identification system, an algorithm adaption should be chosen carefully to give the desired outcome. In this work, the FIR filter is designed with an adaptive plan as said by the structure presented in Fig. 2 [5], [6].

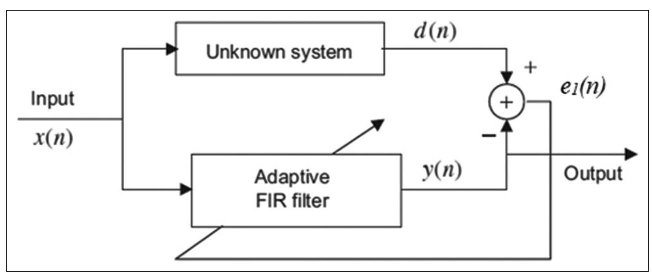

Fig. 1. Adaptive modeling system.

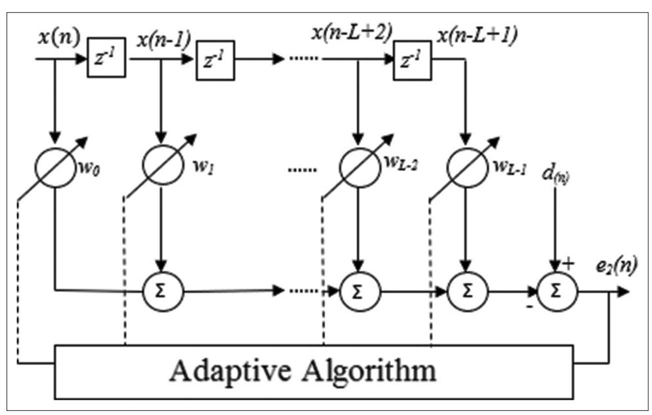

Fig. 2. Structure of adaptive finite impulse response filter.

As part of this research, additional noise (a white noise) has been supplied to the signal $x(n)$, then the input could be displayed as a row vector:

$X(n)=[x(n),x(n-1)\ldots x(n-L+1)]$, n is considered as the time index, L is filter length.

Next, the filter weights $w(n)$ can be represented as follows:

$W(n)=[w_0(n),w_1(n),w_2(n)\ldots w_{L-1}(n)]^T$

The input $X(n)$ is filtered into $e_2(n)$ as output that is produced accordingly by calculating the difference of the adaptive output with the wanted signal, $y(n)$ and $d(n)$, respectively.

The practical sharing piece of the adaptation used an algorithm that is to change the filter weights with length (L) in an iterative method, the weights are changed according to that algorithm every iteration.

This paper employs three different adaptive algorithms to design filter as adaptive with eight as a length to achieve the minimum output error $e_1(n)$ and $e_2(n)$ with the fastest convergence time and lower steady-state misadjustment.

For the mentioned specified modeling system, we are used AAE-ASSLMS adaptive algorithm. To the best of our knowledge, there exist no approaches that aim to get some conclusion or expose a performance result of using this algorithm. For that matter, we have used two other traditional algorithms to make comparisons of the results.

3.1. LMS Adaptive Algorithm

LMS is considered a familiar algorithm that is broadly used because of simple computations and implementation. The filter weights are changed adaptively in harmony with the algorithm at each iteration following the formula [12]:

$$W_{(n+1)}=W_{(n)}+2\mu e_{(n)}X_{(n)} \tag{1}$$

The step-size parameter μ is the lead player in this formula. To achieve the choicest level of convergence, μ is restricted as a small positive value. The step-size value is still fixed for every iteration despite the weights changing which is considered later as a disability in that algorithm with regard to the misadjustment and time convergence balancing issue. With eight weights to be adjusted and a suitable step-size value, FIR AF was successfully designed and implemented in an identification system to identify a BPF.

3.2. NLMS Adaptive Algorithm

NLMS is considered as the first development or expansion of the algorithm of LMS; practically, both algorithms' implementation is alike. The main intention of this extension is to bypass the convergence issue by making the step-size value depending on the recent input signal values with each iteration, in another word, μ is invulnerable to the changes of that input; therefore, the readjustment is not a demand when such changes occur. As the following equation present [20]:

$$\mu(n)=\frac{\mu}{x^T(n)x(n)} \tag{2}$$

Then, the calculated $\mu(n)$ is applied in Eq. 1 for weight adjustment of filter weights. With identical filter length, the adaptation is applied, the outputs fully will be displayed in the result section.

3.3. AAE-ASSLMS Adaptive Algorithm

Absolute average error-variable step-size LMS (AAE-ASSLMS) is an adjusted method of that algorithm of LMS. A VSS is employed in this algorithm which is adjusted in each iteration along with the filter taps (weights). The adjustment of the VSS is established in means of error relying on the recent and previous and calculating the absolute average value as follows [21]:

$$\mu(n+1)=\mu(n)-\left|\sum_{c=0}^{L} e(\mathrm{n}-\mathrm{c})*\beta\right| \tag{3}$$

Where $0<\beta<1$

And

$$\mu(n+1)=\begin{cases}\mu_{min}, & if \quad \mu(n+1)<\mu_{min}\\ \mu(n+1) & otherwise\end{cases} \tag{4}$$

This algorithm efficiently contributes to convergence enhancement by starting with a high value of step-size progressively walking through a reduction process as Eq. 3 illustrates. The current step size $\mu(n)$ will be changing accordingly into $\mu(n+1)$ with dependency on the difference of the absolute average value of the recent and previous error until $\mu(n)$ will reach its minimum in the order of low level of misadjustment achievement and the convergence time speed is acceptable and improved through this transition phase smoothly. By this process, $\mu(n+1)$ will maintain a value less than $\mu(n)$ to maintain the optimum system operation as displayed in Eq. 4, also, the equation clarifies that the adapted step size always retains to μ_{min} to prevent $\mu(n+1)$ from going under the bounded level for the filter stability reservation.

The (β) parameter from Eq. 3 is a factor used to keep convergence time in the phase of the steady state under control while the misadjustment level is controlled by preserving the step-size value by μ_{min}. The value β has chosen in between of $0< \beta < 1$ [21].

The filter weights are now updated based on the following modified formula in preparation for the upcoming iteration.

$$W_{(n+1)} = W_{(n)} + 2\mu_{(n)}\, e^2_{(n)}\, X_{(n)} \tag{5}$$

The convergence factor μ is modified into $\mu(n)$ which can be determined as in Eq. 3 with each iteration will be changed until the algorithm would reach the steady state then the adaptive value reaches its minimum and is reserved at that value to concur with the optimum system operation and maintain its stability with the lowest level of misadjustment.

4. EXPERIMENTAL RESULTS

An adaptive system modeling employing an adaptive FIR filter proposed in this work, designed, and implemented using MATLAB, the AF is designed using three different adaptive algorithms, Fixed step-size LMS, variable step-size NLMS, and adjusted step-size AAE-ASSLMS. The approach parameters are used the same for all the three adaptive algorithms of equal comparison among the output results.

The FIR filter is designed with a length of eight taps, the input for both the indefinite system and the adapted filter is a companion with a white noise signal having zero mean and a unity variance with the main input of SNR (0 dB).

Step-size value selection is set accordingly with the performance and the simulation results examination of the adaptive algorithm AAE-ASSLMS [21]. Of comparison of system performance using the three different adaptive algorithms, the same step value is employed for the other algorithms also.

The filter taps are chosen randomly to show the power of the process of adaptation. The adaptation performance is measured on calculating the normalized difference within the actual taps and the updated ones with a step size equal to 0.05. For a purpose of evaluation, the efficiency of each algorithm and for comparison later, a weight difference (WD) is calculated between the actual and adaptive taps of the filter, as a preparation step for MSE calculation which is aimed at accurate measurement of the resultant output. MSE is calculated for the $e_2(n)$ value in Fig. 2 and according to that value, a comparison will take place between the algorithms for the optimum accuracy to the system performance achievement.

4.1. Simulation of AF

This section reveals the performance evaluation of all three adaptive algorithms that have been employed in the design filter. Finally, the section is concluded by comparing these performance results to illustrate the quality of the proposed model.

4.1.1. LMS performance evaluation

For LMS, step size is fixed and the updated weights are calculated according to Eq. 1. The results are shown in Fig. 3 which demonstrates the outcome of the applying LMS algorithm for both MSE and WD which are scaled according to the output from –20 to 10 dB and –15 to 5 dB, respectively. The outcomes from the initial point are at their peak, then after around 50 iterations, the value of MSE and WD decreases sharply by around 20 and 15 dB, respectively, until gets to their steady-state phase along with the 1000 iterations with fluctuation values –8 (±5) dB for MSE and –9 (±2) for DW, and the recorded minimum values are –13 and –11 dB, respectively. On the other hand, Fig. 4 shows differences between the actual and the evaluated adapted taps, it highlights the performance of the AF in terms of weight adaptation, the figure illustrates how the AF gives weights' values so close to the actual one, while the calculated error is presented with details in Fig. 4 which is the main factor used in the adaptation process.

4.1.2. NLMS performance evaluation

NLMS step size is variable, with regard to this algorithm, the filter taps are updated and weights have gotten from Eq. 2, with the same main parameters, the MSE of output along with WD is illustrated in Fig. 5. Indeed, the figure depicts the minimum recorded values than LMS for both MSE and WD which are –17 and –14, respectively, and the AF reached its steady-state phase with less number of iterations

NLMS results show obvious improvement over LMS results in both MSE and WD, as a consequence of the adaptive formula for step-size value changing, it was the first step for upcoming formulas of system output enhancement.

4.1.3. AAE-ASS performance evaluation

The step size in the AAE-ASS algorithm is adjusted with each iteration following Eq. 3 to give an output represented with both MSE and WD illustrated in Fig. 6. The optimum value of μ_{max} and μ_{min} for the AAE-ASSLMS was chosen to be 0.05 and 0.0005, respectively. The constant β is chosen to be 0.00018 according to both performance analyses.

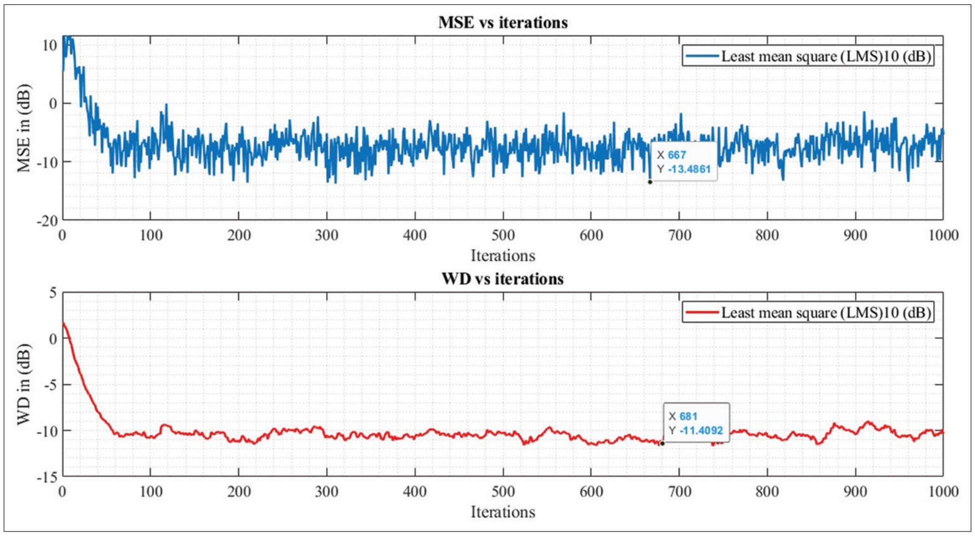

Fig. 3. Least mean square adaptive algorithm results.

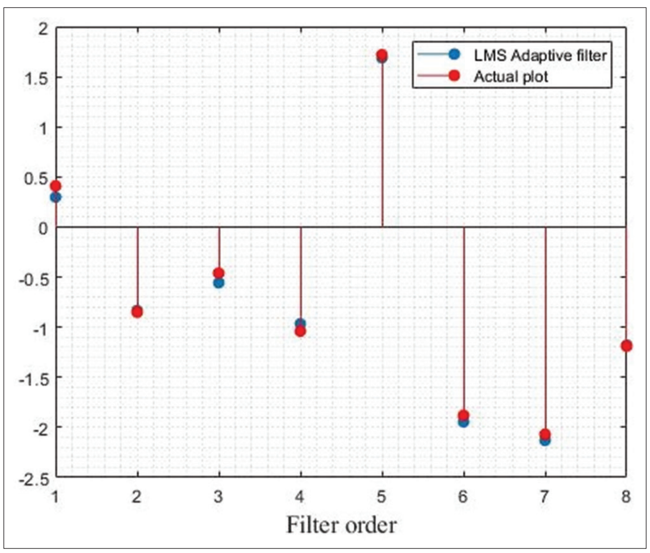

Fig. 4. The actual values of coefficients.

Fig. 6 illustrates the implementation output of the algorithm. Along 1000 iterations, the minimum and maximum recorded MSE are 8 and –46, respectively. In addition to WD values, the recorded output is –8 dB as the initial point then smartly drops to its steady state with –24 dB as its minimum value with noticeable improvement of reaching steady-state phase with a minimum number of iterations.

4.1.4. Comparison result analysis

From Figs. 3 to Fig. 7, minimum MSE has been recorded for each algorithm beside WD as well, Table 1 represents those records for comparison about the accuracy, speed of convergence, and misadjustment. Fig. 4 represents a chart to illustrate the differences in performance among the used algorithms that mentioned in Table 1.

NLMS algorithm has the advantage over the LMS in the means of MSE and weight difference, while the AAE-ASSLMS algorithm achieves the greatest results out of them together. A performance summary for all the algorithms is displayed in Table 1 and illustrated in Fig. 7 for more clarity. Besides, the AAE-ASSLMS algorithm attains the best convergence time among the further algorithms as Fig. 6 illustrates. Furthermore, Fig. 8 shows the three algorithms' performances together, the progress of the AAE-ASS algorithm is noticeable over the further algorithms with regard to convergence speed and steady-state misadjustment.

For further elaboration, the MSE results of both LMS and AAE-ASSLMS algorithms were combined into a singular frame as they are shown in Fig. 9. The MSE in dB for the AAE-ASSLMS algorithm is between around 5 and –20 from the beginning and it has decreased to 0—45, while the MSE for LMS is higher accordingly by around 10 dB along with the iterations.

Despite NLMS algorithm progress over LMS, it was not noticeable to be added to Fig. 9 comparing to the achievement of AAE-ASSLMS algorithms over them both.

4.2. Simulation of Adaptive System Identification through a BPF

In this part, the MATLAB program is employed to show the filter adaption and can keep tracking the behavior of the mysterious system which is set to be a BPF whose cutoff

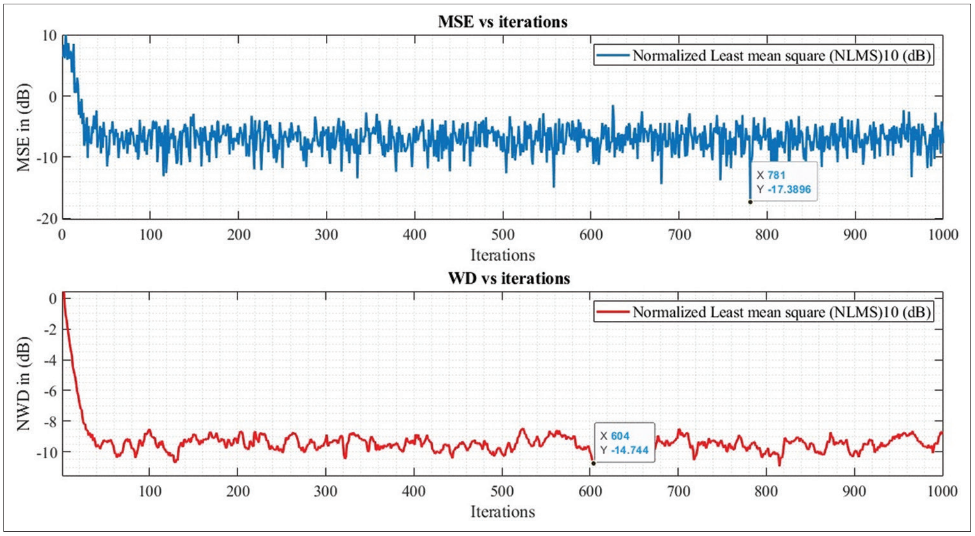

Fig. 5. Normalized least mean square adaptive algorithm results.

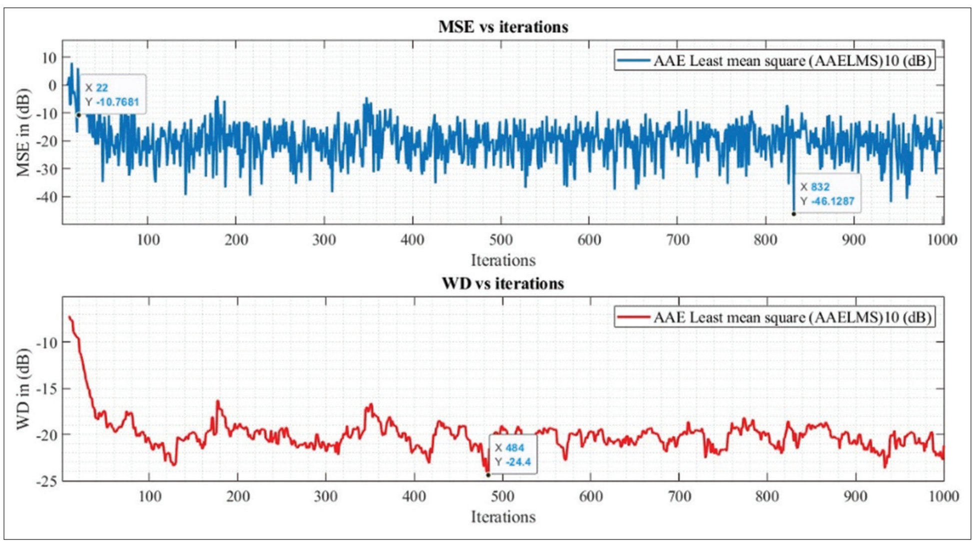

Fig. 6. Absolute average error-based adjusted step-size least mean square adaptive algorithm results.

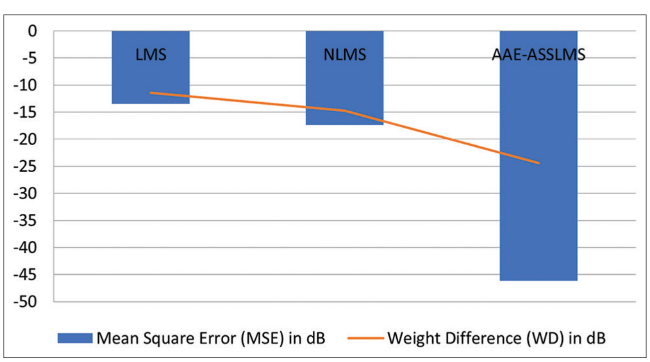

Fig. 7. Summary of adaptive algorithms performance.

frequencies are 1500 Hz and 1700 Hz operating at 8000 Hz. The input consists of tones of 600, 1600, and 2600 Hz. The input with its spectrum is presented in Figs. 10 and 11.

The assumed BPF frequency response is displayed in Fig. 12.

Predictably, the output response of the system should be 1600 HZ tone only since the other tones are rejected by the filter. However, the designed filter of this work along with the three different mentioned algorithms should follow the BPF output. Fig. 13 depicts the frequency domain comparisons; it

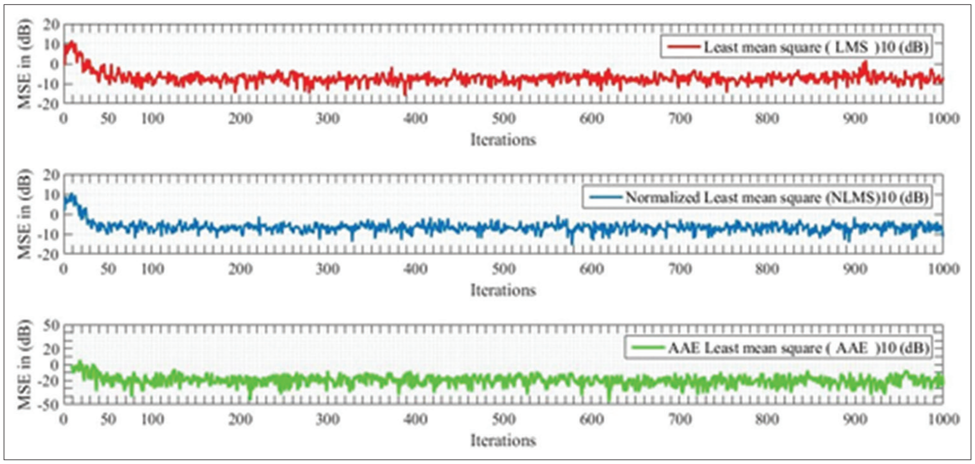

Fig. 8. Algorithms' performance.

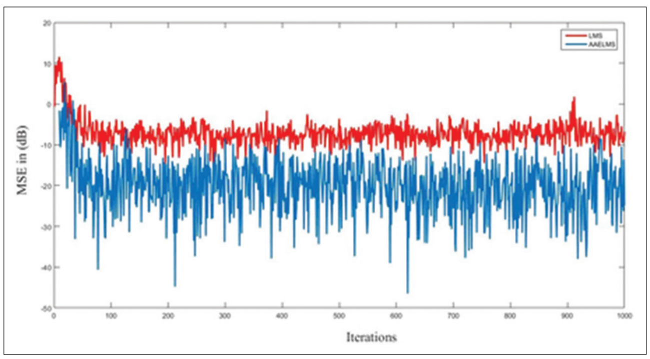

Fig. 9. Mean square error output of least mean square (LMS) and absolute average error-based adjusted step-size LMS algorithms.

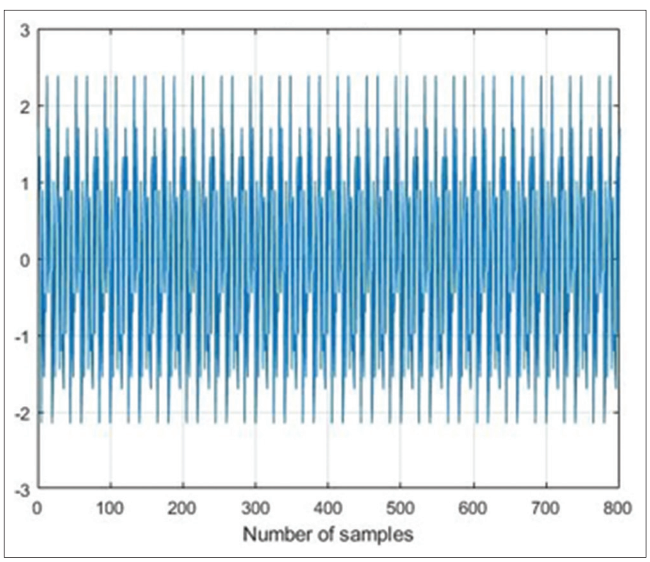

Fig. 10. The input signal to the system.

also reveals the matching spectrum output signal for the BPF and the filter adapted using the AAE-ASSLMS algorithm the

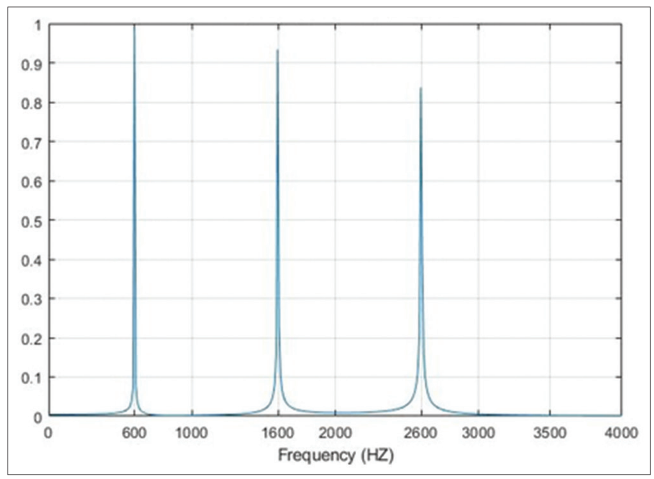

Fig. 11. The spectrum of the input signal.

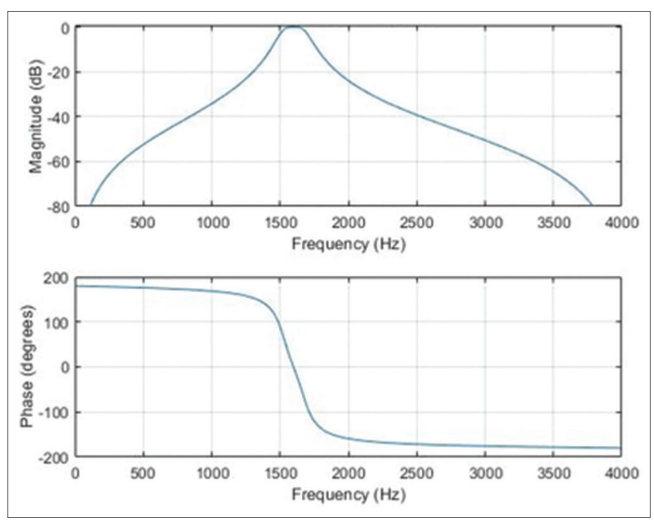

Fig. 12. The frequency response of the unknown system (bandpass filter).

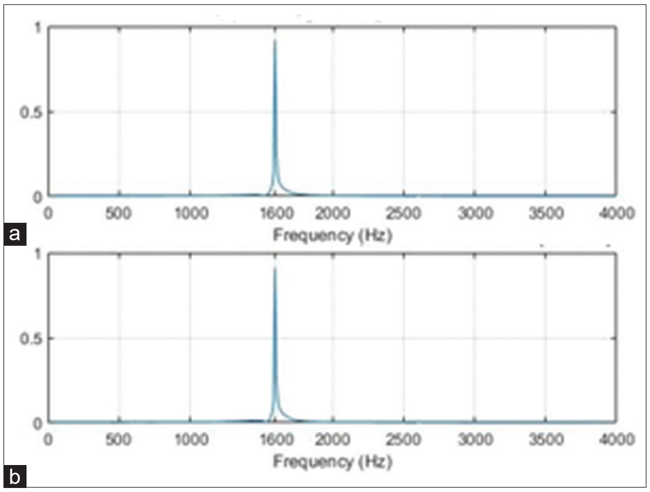

Fig. 13. The spectrum of the output signals for (a) the unknown system (bandpass filter) and (b) the adaptive filter (absolute average error-based adjusted step-size least mean square algorithm).

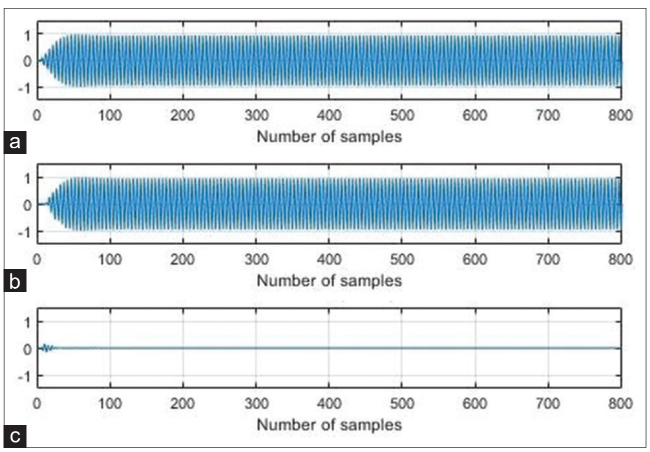

Fig. 14. The output signal of (a) the unknown system (bandpass filter), (b) the adaptive filter (absolute average error-based adjusted step-size least mean square algorithm), and (c) the error signal $e_1(n)$.

output signal spectrum of the three algorithms is identical to 1600 Hz.

Despite the matching spectrums, although an error accrued and calculated ($e_1(n)$ in Fig. 1). Fig. 14 shows the total output signal with the error calculated and presented through the difference between BPF and the design filter output with the (AAE-ASSLMS algorithm) algorithm.

The calculated error for each used algorithm in this study as Fig. 15 illustrated.

Through the simulated output results, it is noticeable that resultant error from the AAE-ASSLMS algorithm is

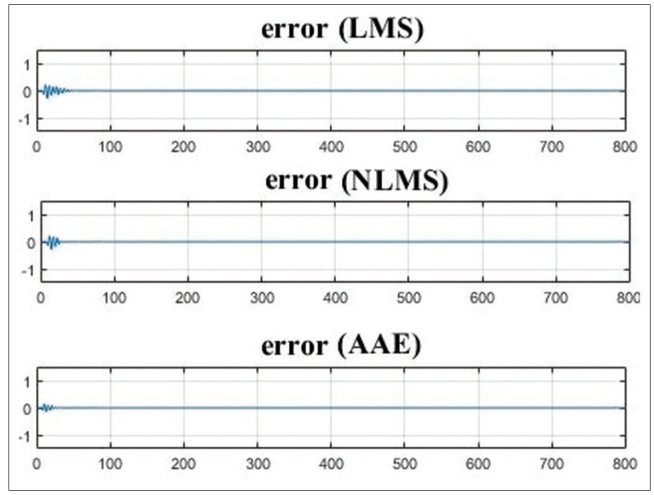

Fig. 15. The error for each algorithm.

TABLE 1: Summary of adaptive algorithms performance

Algorithm	Iteration	Filter order	Mean square error in dB	Weight difference in dB
LMS	1000	8	–13.4861	–11.4092
NLMS	1000	8	–17.3896	–14.744
AAE-ASSLMS	1000	8	–46.1287	–24.4

the minimum in the mean of peak-to-peak value and the system identification time is the shortest among the other algorithms in means of the samples number. In another word, the designed filter identified the indefinite system with <10 samples, meanwhile, other algorithms were taking more than 20 samples for identification.

5. CONCLUSIONS

Two significant parameters that are directly impacted by the step size in VSS algorithms, there are the low-level misadjustment in the steady-state actions and the convergence time speed in the transient actions. The relation between step-size, MSE, and the convergence speed is difficult to find a middle ground to be operated on because of the direct and inverse proportions among them (direct with MSE, inverse with convergence speed). A VSS algorithm is employed within this paper called the absolute average error adjusted step-size LMS algorithm (AAE-ASSLMS) to design an adaptive FIR filter along with LMS and NLMS algorithms. The adaptation step-size value is adjusted based on the absolute and average value for the recently assessed error and the prior one, the adjusting process in the algorithm takes place through maximum down to minimum step-size selection for optimum achievement in means of fast convergence time and the lowest level of misadjustment. Throughout the simulated results, the algorithms' output shows improvement in those parameters compared with fixed step-size LMS and its modified version (NLMS). Within this work, the designed filter with the AAE-ASSLMS algorithm is employed for the 1st time in a modeling system for higher accuracy of identification. Furthermore, to prove the system efficiency improvement, a BPF is employed in this paper as a system to be identified, successfully, the identification process achieved using the designed adapted FIR filter through the different used algorithms. Again, the outputs of the simulation of the (AAE-ASSLMS) algorithm show improvement in the performance with regard to error and identification time in means of samples numbers over the other algorithms that are used in adaptation within the same parameters.

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